Electromagnetic Interaction of Complex Scalar Fields

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The complex scalar fields may interact with electromagnetic fields since one can construct the gauge invariant Lagrangian density. However, it is shown that the current of the scalar fields becomes gauge dependent, and thus the fields should not be physical observables. Further, we study the gauge invariant Lagrangian density for the scalar fields which couple to the non-abelian gauge fields and show that the color singlet current of the scalar fields is also gauge dependent and thus there is no chance that the scalar fields become observables.

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I. INTRODUCTION

II. COMPLEX SCALAR FIELDS

In field theory text books, it is most common that people start to discuss the scalar field theory, and they construct the Lorentz invariant Lagrangian density of scalar field. Here, we first write the Lagrangian density of scalar field with its mass m as

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \frac{1}{2} m^2 \phi^\dagger \phi \tag{2.1}$$

where ϕ is defined as the sum of two real scalar fields $\phi_1,\ \phi_2$

$$\phi = \phi_1 + i\phi_2. \tag{2.2}$$

From the Lagrange equation, one can find the following Klein-Gordon equations

$$\partial_\mu \partial^\mu \phi_i - m^2 \phi_i = 0, \qquad (i = 1, 2). \tag{2.3}$$

This is a free particle state, and thus there is no way to observe it. Therefore, we should find out interactions with electromagnetic fields. The way to construct the gauge invariant Lagrangian density is well known, and we will discuss it below.

III. SCALAR FIELD WITH GAUGE FIELD

The Lagrangian density of scalar fields interacting with gauge fields can be given as

$$\mathcal{L} = \frac{1}{2}(D_\mu \phi)^\dagger (D^\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{3.1}$$

where

$$D_\mu = \partial_\mu + ieA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{3.2}$$

Here ϕ denotes a complex scalar field and A_μ is a vector potential. e denotes the gauge field coupling constant which shows the strength of the interaction.

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A. Gauge Invariance

Here we should examine the gauge invariance of the Lagrangian eq.(3.1). The gauge transformation is defined here as

$$\begin{aligned} A'_\mu &= A_\mu + \partial_\mu \chi \\ \phi' &= e^{-i\chi}\phi. \end{aligned} \tag{3.3}$$

Now we insert this transformation into $D^\mu \phi$ and we see

$$D'_\mu \phi' = [\partial_\mu + ie(A_\mu + \partial_\mu \chi)]e^{-ie\chi}\phi = D_\mu \phi \tag{3.4}$$

which is invariant under the gauge transformation.

B. Noether Current

From the Lagrangian density of eq.(3.1), one can obtain the Noether current J^μ

$$J^\mu = \frac{1}{2} i \left\{ \phi^\dagger (\partial^\mu + igA^\mu)\phi - \phi(\partial^\mu - igA^\mu)\phi^\dagger \right\} \tag{3.5}$$

and the gauge invariance of this current density J^μ is guaranteed. Also, one can check that the current density J^μ is conserved, that is

$$\partial_\mu J^\mu = 0. \tag{3.6}$$

Therefore the current density J^μ is gauge invariant and it is conserved.

C. Current Density j_s^μ of Scalar Field

However, the current density j_s^μ of the scalar field is different from J^μ and is written as

$$j_s^\mu = \frac{1}{2} i \left\{ \phi^\dagger \partial^\mu \phi - \phi \partial^\mu \phi^\dagger \right\}. \tag{3.7}$$

Now a question may arise as to whether this j_s^μ should be invariant under the gauge transformation of eq.(3.3) or not. Indeed one sees that this current density j_s^μ of scalar field is not gauge invariant since

$$j'^\mu_s = j_s^\mu + e\phi^\dagger \partial^\mu \chi \phi. \tag{3.8}$$

It is now important to realize that the current density of scalar field should not be observed. This means that the charge of the scalar field should not be observables.

IV. SCALAR FIELD WITH NON-ABELIAN GAUGE FIELD

Now we should extend the formulation of the scalar field coupling to the U(1) gauge field to the non-abelian gauge field. In this case, the Lagrangian density of scalar fields interacting with non-abelian gauge fields can be given as

$$\mathcal{L} = \frac{1}{2}(D_\mu \phi)^\dagger (D^\mu \phi) - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} \tag{4.1}$$

where

$$D_\mu = \partial_\mu + ig\boldsymbol{A}_\mu \cdot \boldsymbol{\tau}, \quad \boldsymbol{G}_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + ig[\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]. \tag{4.2}$$

Here, the SU(2) generator $\boldsymbol{t} = \frac{1}{2}\boldsymbol{\tau}$ should satisfy the following relation

$$[t^a, t^b] = iC^{abc}t^c \tag{4.3}$$

where C^{abc} denotes the structure constant, and $\boldsymbol{\tau}$ is the Pauli matrix. ϕ denotes the two component spinor which should be written as

$$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} \tag{4.4}$$

In this case, the Lagrangian density eq.(4.1) is invariant under the gauge transformation of

$$\boldsymbol{A'}_\mu = \boldsymbol{A}_\mu + \partial_\mu \boldsymbol{\chi} \tag{4.5}$$

$$\phi' = e^{-ig\boldsymbol{\chi}\cdot\boldsymbol{\tau}}\phi. \tag{4.6}$$

Here, we make use of the following identity

$$e^{-ig\boldsymbol{\chi}\cdot\boldsymbol{\tau}} = \cos g\chi - \frac{i}{\chi}(\boldsymbol{\chi}\cdot\boldsymbol{\tau})\sin g\chi \tag{4.7}$$

$$e^{ig\boldsymbol{\chi}\cdot\boldsymbol{\tau}}e^{-ig\boldsymbol{\chi}\cdot\boldsymbol{\tau}} = \left(\cos g\chi + \frac{i}{\chi}(\boldsymbol{\chi}\cdot\boldsymbol{\tau})\sin g\chi\right)\left(\cos g\chi - \frac{i}{\chi}(\boldsymbol{\chi}\cdot\boldsymbol{\tau})\sin g\chi\right) = 1. \tag{4.8}$$

Now the color singlet scalar current j_μ

$$j_\mu = \frac{1}{2}i\left\{(\phi)^\dagger\partial_\mu\phi - \phi\partial_\mu\phi^\dagger\right\} \tag{4.9}$$

becomes gauge dependent since

$$j'^\mu = j^\mu + g\phi^\dagger\partial^\mu(\boldsymbol{\chi}\cdot\boldsymbol{\tau})\phi. \tag{4.10}$$

Therefore, it should be important to realize that the scalar current is gauge dependent and thus it is not a physical observable even though it is a color singlet current. It should be noted that the color current such as quark or gluon currents are gauge dependent, and thus they are not physical observables and this is just the confinement of quarks or gluons in QCD.

V. EIGENSTATE OF CHARGE

Here, a question may arise as to what is the charge of the scalar field. In order to clarify the concept of charge, we should define the charge operator as

$$\hat{Q}|q\rangle = q|q\rangle \tag{5.1}$$

where q takes $q = \pm 1$. The charge zero state cannot interact with the gauge field and thus we should exclude the charge zero state in this scheme.

In addition, the charged state should not necessarily interact with the electromagnetic field. There is a good example which is weak bosons of $W^\pm$. The charge of $W^\pm$ should be

$$\hat{Q}|W^\pm\rangle = \pm|W^\pm\rangle. \tag{5.2}$$

However these bosons cannot interact with the electromagnetic field since the weak currents of $W^\pm$ bosons cannot couple with the electromagnetic field. Instead, they interact with electrons and neutrinos.

A. Charge Quantum Number

In this respect, the charge should be considered as the quantum number of particles. Therefore, it should be a conserved quantity. At the present stage, this conservation should be valid rigorously, and if there is any violation of this charge quantum number, then there must be a proper reason of the violation.

B. Charge State of Complex Scalar Field

If we define the complex scalar field by

$$\phi = \phi_1 + i\phi_2 \tag{5.3}$$

then this must be an eigenstate of the charge operator, and thus

$$\hat{Q}|\phi^+\rangle = +|\phi^+\rangle. \tag{5.4}$$

Therefore ϕ_1 and ϕ_2 must satisfy

$$\hat{Q}|\phi_1^+\rangle = +|\phi_1^+\rangle, \quad \hat{Q}|\phi_2^+\rangle = +|\phi_2^+\rangle. \tag{5.5}$$

In this case, we do not understand the meaning of the label 1 and 2 in the ϕ_1 and ϕ_2 states.

VI. CHARGE TRANSFER IN HIGGS MECHANISM

In the Higgs mechanism, the charge of the scalar boson should be transferred to the electromagnetic field. This is quite mysterious and we should clarify as to what should have happened to the charge transfer in the Higgs mechanism.

A. Higgs Mechanism

Before discussing the transfer of the charge, we briefly review the Higgs mechanism. The Lagrangian density is given as

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi)^\dagger(D^\mu\phi) - \frac{1}{4}u_0\left(|\phi|^2 - \lambda^2\right)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{6.1}$$

where u_0 and λ are constant. The second part of the Lagrangian density denotes the Higgs potential which is introduced by hand. Now the equations of motion for the scalar field ϕ become

$$\partial_\mu(\partial^\mu + igA^\mu)\phi = -u_0\phi\left(|\phi|^2 - \lambda^2\right) - igA_\mu(\partial^\mu + igA^\mu)\phi \tag{6.2}$$

$$\partial_\mu(\partial^\mu - igA^\mu)\phi^\dagger = -u_0\phi^\dagger\left(|\phi|^2 - \lambda^2\right) + igA_\mu(\partial^\mu - igA^\mu)\phi^\dagger. \tag{6.3}$$

On the other hand, the equation of motion for the gauge field A_μ can be written as

$$\partial_\mu F^{\mu\nu} = gJ^\nu \tag{6.4}$$

where

$$J_\mu = \frac{1}{2}i\left\{\phi^\dagger(\partial_\mu + igA_\mu)\phi - \phi(\partial_\mu - igA_\mu)\phi^\dagger\right\}. \tag{6.5}$$

One can also check that the current J_μ is conserved, that is

$$\partial_\mu J^\mu = 0. \tag{6.6}$$

B. Unitary Gauge Fixing

In the Higgs mechanism, the central role is played by the gauge fixing of the unitary gauge. In the unitary gauge one takes

$$\phi = \phi^\dagger \tag{6.7}$$

which means that $\phi_2 = 0$ from eq.(5.3). This is quite a strange result from the definition of the complex scalar field since this gauge fixing has nothing to do with the gauge field of A^μ. In any case, this is the constraint on the scalar field ϕ even though there is no gauge freedom in this respect. For the scalar field, the phase can be changed, but this does not mean that one can erase one degree of freedom. One should transform the scalar field in the gauge transformation as

$$\phi' = e^{-ig\chi}\phi \tag{6.8}$$

but one must keep the number of degree of freedom after the gauge transformation. Whatever one fixes the gauge χ, one cannot change the shape of the scalar field ϕ since it is a functional variable and must be determined from the equations of motion. The gauge freedom is, of course, found in the vector potential A_μ as we discussed above. In this sense, one sees that the unitary gauge fixing is a simple mistake. The basic reason why people overlooked this simple-minded mistake must be due to their obscure presentation of the Higgs mechanism. Also, it should be related to the fact that, at the time of presenting the Higgs mechanism, the spontaneous symmetry breaking physics was not understood properly since the vacuum of the corresponding field theory was far beyond the proper understanding. Indeed, the Goldstone boson after the spontaneous symmetry breaking was taken to be almost a mysterious object since there was no experiment which suggests any existence of the Goldstone boson. Instead, a wrong theory prevailed among physicists. Therefore, they could assume a very unphysical procedure of the Higgs mechanism and people pretended that they could understand it all.

C. Final Lagrangian Density

After an improper gauge fixing, one arrives at the final Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \frac{1}{4}u_0\left(|\lambda+\eta(x)|^2 - \lambda^2\right)^2 + \frac{1}{2}g^2(\lambda+\eta(x))^2 A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{6.9}$$

where we rewrite the Higgs field as

$$\phi = \phi^\dagger = \lambda + \eta(x). \tag{6.10}$$

Since the real scalar field η is supposed to be small and besides a real scalar field is unphysical, it may be set to zero, that is, $\eta = 0$. In this case, we arrive at the following Lagrangian density

$$\mathcal{L} = \frac{1}{2}g^2\lambda^2 A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{6.11}$$

This should be the final Lagrangian density of the Higgs theory, and it is nothing but the massive vector boson field which has nothing to do with the gauge theory.

D. Transfer of Charge

The original scalar field ϕ is a complex field, and it is written as

$$\phi = \phi_1 + i\phi_2 \tag{6.12}$$

VII. CONCLUSIONS

In this short note, we have presented the intrinsic problem of minimal transformation for the complex scalar fields. The fact that the minimal transformation should not be taken as a principle may well be well-known to educated physicists. In this respect, this short note is only to confirm that the minimal transformation must be taken as a result of the gauge transformation.

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[1] J.D. Bjorken and S.D. Drell, "Relativistic Quantum Mechanics", (McGraw-Hill Book Company,1964)
[2] J.J. Sakurai, "Advanced Quantum Mechanics", (addison-Wesley,1967)
[3] K. Nishijima, "Fields and Particles", (W.A. Benjamin, INC, 1969)
[4] T. Fujita and N. Kanda, "Fundamental Problems in Quantum Field Theory" (Bentham Publishers, 2013)