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Physics Not Even Special

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Contents

Chapter 1. Theory of Relativity

The aim of physics is to understand nature. All the theoretical models should be constructed so as to clarify physical phenomena, and that is all. Therefore, any theoretical model should not lose its aim of understanding nature. At present, all the basic field theory models are quite normal, except the general relativity. The Einstein equation has no corresponding physical phenomena since it is constructed for the coordinate system whose physical meaning is vague.

1.1 Space

Physical phenomena occur in space. Most people may have thought about what space should be. However, we cannot realize space itself unless something should exist in front of us. In physics, we introduce a coordinate system in which $x-$axis, $y-$axis and $z-$axis should be defined. This coordinate system represents space and we describe nature in this coordinate system.

1.2 Coordinate System

In physics we introduce the coordinate system in space and intend to explain the motion of a particle. The motion of a particle is described by the coordinate (x, y, z) and often we write it as $\boldsymbol{r} \equiv (x, y, z)$ for convenience. But in field theory calculation, we would rather use the component representation of x_i since this is more useful than the vector representation.

1.2.1 Origin of Coordinate System

In the coordinate system, one should determine the origin. Where should it be the best? If one hesitates where to decide the origin of the coordinate system, this is quite a normal reaction. For example, if one wishes to describe the motion of the earth around the sun, then it is reasonable that one chooses the origin at the center of the sun. In reality, it is indeed simpler if one chooses the origin at the center of the sun, but practically one can fix the origin of the coordinate system at any point of space. This is because there is a translational invariance for any physics law. Therefore, the motion of a particle does not depend on the choice of the origin of the coordinate system. As a result, one can choose the origin of the coordinate system as one likes, and thus, it is better if one chooses the origin such that the description of the particle motion becomes simpler. For example, if one treats the earth motion around the sun, then one can take the origin at the center of the sun.

1.2.2 Cartesian Coordinate

The coordinate system is the essence in mathematics. The basic coordinate system is cartesian coordinate (x, y, z) which is most simple, and this must be the starting point of the coordinate system.
However, many types of potentials in physics can be written in terms of polar coordinate, and therefore, when one wishes to solve the equation of motion, one should better make use of the polar coordinate representation.

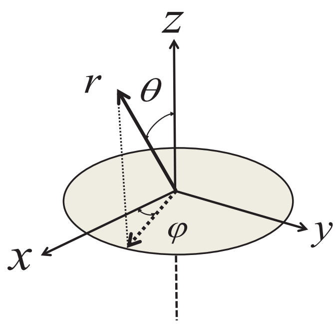

Figure 1.1: Coordinate System

- **Polar Coordinate :** (r, θ, φ) $\qquad \begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdydz = \int_0^{\infty} r^2 dr \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi$$

1.3 Time

What is "time"? This is a very difficult question, and indeed physicists cannot answer for this question. Time is measured by the earth rotation around the sun or frequency of photon emitted from atom (Cs atom), but this does not mean that we understand what time should be. At present we make a time series of second, minute etc. and from this time series, we measure and recognize time. This is all that we can do concerning time.

Time flows homogeneously, and this is an assumption. However, there is no incident which disagrees with this assumption. The homogeneity of time is related to the energy conservation, and therefore, we believe that the homogeneous flow of time should be valid exactly. It should be noted that the homogeneity of space is related to momentum conservation.

1.3.1 Origin of Time

How should we choose the origin of time when we want to solve the equation of motion? The answer is rather simple. One can choose the origin of time as one wishes at the starting point. Time development of a particle should be measured from the starting point of the observer as he likes.

1.3.2 Time Dependence

In physics, the time dependence of physical quantity must be very important. If a physical quantity does not depend on time, it should stay in the same state all the time, and thus it is called "conserved quantity". For example, the orbit of the earth around the sun is on the plane, and the reason why it is on the plane can be closely related to the conservation of angular momentum. Due to the conserved quantity of angular momentum, one can fix the direction of angular momentum at $z-$axis. Since the motion of the earth must be orthogonal to the angular momentum $\boldsymbol{L}$, one can prove that the motion of the earth must be on the $x - y$ plane.

1.4 Basics of Relativity

Basic equations in physics intend to describe particle motions in time and space. As discussed previously, we introduce the coordinate system for the description of particle motion. Here, there is the most important law of nature which is called "relativity principle". Now we assume that A-system is moving with a constant velocity of $\boldsymbol{v}$ relative to B-system. In this case, all the physical observables must be the same both in A and B systems, and this is due to the relativity principle. Here one can easily understand the importance of this law since any theoretical framework developed on the earth should be also valid on the other stars of the universe. Otherwise there is no meaning of hard works here to construct the theory in physics on the earth, and therefore, this requirement is absolutely necessary. In fact, there is no phenomenon in nature which disagrees with the relativity principle after many years of vast amount of experiments and data analysis.

In this case, what kind of relations should we require for A and B systems in order to satisfy the relativity principle? This is, of course, quite well known. and equations of motion written in A-system should be transformed into the same shape of equations of motion in B-system in terms of Lorentz transformation. The mathematical space of this transformation is called Minkowski space, but it should be noted that this space has nothing to do with four dimensional space. Our physical space is 3 space dimensions plus time, and this is all. In fact, the mathematical meaning of Lorentz transformation should be to make $s^2 \equiv (ct)^2 - x^2 - y^2 - z^2$ invariant where c denotes light velocity.

1.4.1 Lorentz Transformation

Now we consider the case in which A-system is moving along $x-$axis with the velocity of v relative to B-system. In this case, the transformation property is given by Lorentz transformation. We denote the coordinate of A-system by $S(t', x', y', z')$ while the coordinate of B-system is written as $R(t, x, y, z)$. Time should be different from each other, but this is reasonable since the observer of A-system should have different time from that of B-system. In this case, the Lorentz transformation is written as

$$x = \gamma(x' + vt'), \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right), \quad y = y', \quad z = z' \tag{1.1}$$

where γ is defined as $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. Eq.(1.1) is obtained such that Maxwell equations must be the same between A and B systems. If the velocity v in eq,(1.1) is sufficiently smaller than the velocity of light c, then the Lorentz transformation becomes

$$x \simeq x' + vt', \quad t \simeq t', \quad y = y', \quad z = z' \tag{1.2}$$

which is just the same as the Galilei transformation. In this sense, most of the phenomena occurred on the earth should be safely treated by non-relativistic kinematics. In fact, the fastest object observed on the earth must be the revolution velocity of the earth around the san, and this is $v \simeq 10^{-4}\,c$ which is sufficiently smaller than c.

1.4.2 Importance of Relativity

The Lorentz transformation itself is very simple and it is quite easy to check the transformation property. However, it is non-trivial to understand physics related to the Lorentz transformation. One has to think it over quite in depth what the Lorentz transformation should mean in connection with nature. In any case, the Lorentz transformation is obtained so that the Maxwell equations must have the same shape for any inertial systems as one chooses. In addition, any physical observables must have the same value for any inertial systems, and this is the very important point of the relativity.

In this short lecture, we discuss what should be physical observables and readers should understand the meaning of physical observables in their own words.

1.4.3 Verification of Relativity

Theory of relativity is often explained in terms of science history, and indeed this may also be important. However, we would like to describe the theory of relativity from the point of scientific logics.

At present, all the physical phenomena should be understood in terms of quantum field theory, and any theoretical models in field theory must keep the invariance under the Lorentz transformation. In this sense, relativity is the basis of modern physics and any models that contradict the relativity cannot be accepted for modern physics. Therefore, it is rigorous that there should not be any exceptions. Since the general relativity violated this principle of relativity, it is very hard to accept the Einstein equation as the fundamental equation.

1.5 General Relativity

Almost a century ago, Einstein proposed the general relativity. The Einstein equation is an equation for metric tensor $g^{\mu\nu}$, and it is constructed such that the metric tensor may well be affected by the presence of the star distributions. However, there is no physical reason for this effect on the change of $g^{\mu\nu}$. Further, there is no requirement from nature, and simply it is a theoretical assumption.

1.5.1 Super Simplified Einstein Equation

Here we present a simplified Einstein equation in which space is assumed to be one dimension. Therefore, the equation becomes for $g_0(x)$, $g_1(t)$ as unknown functions. Here we write the Einstein equation for $g_0(x)$

$$\frac{d^2 g_0(x)}{dx^2} = 2G\varrho_0(x) \tag{1.3}$$

where $\varrho_0(x)$ denotes the distribution of stars. This equation has the same structure as the four dimensional Einstein equation.

• **Coordinate System of Distribution Function :** Here, a question should arise as to what is the coordinate of $\varrho_0(x)$? This may well be defined as the old coordinate system which is before the effect of the star distribution in the equation. But it is clear that this violates the causality.

1.5.2 General Relativity and Gravity

The general relativity is considered to be a theory of gravity. This is, however, an optimistic assumption, and in fact, there is no way to prove that the Einstein equation is related to the gravity. Why did people believe that the general relativity is related to the gravity? This is due to the following hypothesis for the metric tensor

$$g_0(x) \simeq 1 + 2\phi_g. \tag{1.4}$$

If this equation were valid, then one would have obtained the gravitational Poisson equation which can produce a proper gravitational potential. However, the metric tensor is an unknown function which should be determined as a solution in the Einstein equation. Therefore, it is clear that there is no way to assume any shape of the metric tensor before solving the Einstein equation.

1.5.3 General Relativity Violates Relativity Principle

As can be seen from eq.(1.3), the metric tensor becomes coordinate dependent, and thus, the general relativity violates the relativity principle. In this respect, it is clear that the general relativity cannot be accepted as a fundamental theory in modern physics.

1.5.4 Gravity Theory in Future

We prove that the general relativity has nothing to do with the gravity. In this case, people may well be worried about what should be done for the gravity theory. Fortunately, the new gravity model is already proposed in terms of field theory terminology, and this new theory can explain all the experiments and observed data concerning the gravity. [4, 5].

1.6 Homework Problems

Answer the following questions with your own words.

1.1 Using the Lorentz transformation (1.1), confirm that s^2 is the same for any systems. For this, you should prove $s^2 = s'^2$.

1.2 As the differentials of coordinate $x^\mu = (x_0, x_1, x_2, x_3)$, we introduce

$$dx^\mu \equiv (dx^0, dx^1, dx^2, dx^3) \equiv (cdt, dx, dy, dz) \quad (1.5)$$

$$dx_\mu \equiv (dx_0, dx_1, dx_2, dx_3) \equiv (cdt, -dx, -dy, -dz). \quad (1.6)$$

This way of expressing upper and lower indices is introduced in the textbook by Bjorken-Drell. For example, we see

$$dx_\mu dx^\mu \equiv dx_0 dx^0 + dx_1 dx^1 + dx_2 dx^2 + dx_3 dx^3 \quad (1.7)$$

$$\equiv (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2 \quad (1.8)$$

In this way, if μ appears repeatedly, then the summation of $\mu = 0, 1, 2, 3$ should be taken. Here, Minkowski introduces the metric tensor $g^{\mu\nu}$ as

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2 = g^{\mu\nu} dx_\mu dx_\nu \quad (1.9)$$

In this case, what is the shape of $g^{\mu\nu}$?

1.3 Einstein assumed that metric tensor $g^{\mu\nu}$ should depend on time and space. In this case, what should happen to the Lorentz invariance of $(ds)^2$?

1.4 If a classical wave propagates in vacuum, then this disagrees with the relativity of the system. Those who claim that they discovered the gravitational wave should understand the way in which the classical wave propagates in vacuum. Discuss with your friend about this "discovery".

1.5 Sound wave should propagate through medium such as air or water. Then, what should be a relativity between sound source and observer?

Chapter 2. Physics of Electrons

Physical phenomena in microscopic worlds can be governed by electron motion. The dynamics of electrons is described by quantum mechanics, and therefore, it should be necessary to understand the basic physics of quantum mechanics. There, it should be most important to think of what should be physical observables.

Physical phenomena in the world should be governed by the motion of electrons. In this case, if the macroscopic number of electrons are involved, then the physics laws are determined from the Maxwell equations. But the electric currents appearing there should be described by quantum mechanics, even though the problem of currents is quite complicated due to many body interactions. In this sense, if some phenomena are connected to the many body problem, then it is normally too difficult to handle them properly. In addition, physics of liquid or solution must be very difficult to understand since it should involve the gravitational force of the earth, though indirectly.

2.1 Quantum Physics

Quantum theory can describe the microscopic world, and it is very reliable. Up to now, there is no phenomenon which contradicts quantum theory. This is one of the most important theoretical frameworks together with electromagnetisms in modern physics.

2.1.1 Atom

At the center of atom, there is a nucleus which is composed of protons and neutrons. Among nucleons (proton and neutron), there is a strong interaction which can bind nucleons in nucleus. The binding energy of nucleons should amount to several MeV which should be compared to that of electrons in atom which is several eV.

On the other hand, the main properties of atoms can be determined by electrons bound in atom. The smallest atom must be a hydrogen atom where a proton binds one electron. The interaction between proton and electron is the Coulomb force, and its dynamics should be determined by the Schrödinger equation which is the basic equation in quantum mechanics.

2.1.2 Schrödinger Equation

To our readers of this lecture, it may not be very easy to understand the Schrödinger equation in depth. In particular, if one wishes to solve the Schrödinger equation, then one should make use of many mathematical techniques. Therefore, non-specialists do not have to master the way to solve the Schrödinger equation. However, this equation itself is very important, and thus, some of the ideas and concepts in connection with the Schrödinger equation should be understood so as to train and improve their way of thinking in physics.

In particular, the state vector or wave function must play a central role in quantum mechanics, and therefore it should be necessary to understand the physical meaning of state vector. Usually we denote the state vector of electron by $\Psi(\boldsymbol{r},t)$, and we calculate physical observables such as energy eigenvalue in terms of the state vector in quantum mechanics. The equation for this state vector $\Psi(\boldsymbol{r},t)$ is the Schrödinger equation which is written as

$$i\hbar\frac{\partial\Psi(\boldsymbol{r},t)}{\partial t} = \hat{H}\Psi(\boldsymbol{r},t) \tag{2.1}$$

where $\hat{H}$ is called Hamiltonian. In eq.(2.1), there appears "i" which should guarantee the time reversal invariance ($t \to -t$) of the Schrödinger equation.

If one wishes to treat the bound state problem, then one assumes the following shape for the wave function

$$\Psi(\boldsymbol{r}, t) = e^{-i\frac{E}{\hbar}t}\psi(\boldsymbol{r}) \tag{2.2}$$

and therefore, the equation for $\psi(\boldsymbol{r})$ becomes

$$\hat{H}\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r}), \quad \text{where} \quad \hat{H} = \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + U(r)\right]. \tag{2.3}$$

This is a time independent Schrödinger equation. Here, $\hat{H}$ is assumed to be one particle system, and m denotes the mass of electron. $U(r)$ is a potential which acts on electron. E is the energy of electron, and this is the eigenvalue of the Hamiltonian $\hat{H}$. Here, it should be noted that the Hamiltonian $\hat{H}$ is an hermite operator, and thus its eigenvalue must be always a real number.

If one solves the Schrödinger equation, then one obtains the energy eigenvalue of E as well as the wave function $\psi(\boldsymbol{r})$. In this case, a quantum number n appears and it can specify the wave function and its eigenvalue. Therefore, we often write $\psi_n(\boldsymbol{r})$ or E_n.

• **Origin of Potential $U(r)$:** The potential $U(r)$ is produced by some other object than the corresponding electron. For example, electron in hydrogen atom feels a potential which is produced by proton. In this case, this potential is written as $U(r) = -\frac{e^2}{r}$, and it is called a Coulomb potential which is attractive. Proton is assumed to be at rest, and this is a good approximation since proton mass is about 2000 times larger than that of electron.

• **Quantum Number :** The most important concept in quantum mechanics is “quantum number” which specifies the wave function. Electrons cannot be distinguished from each other, but also there is no reason to make any difference between many electrons.

However, if there are many electrons in atom, it is sometimes important to specify which electron is interacting and active with photons so that the reaction process can be better understood. In this case, if we can specify the quantum number of the electron wave function, then we can calculate the reaction process. We often state that the photon emission is now made by the $1s-$electron or something like that.

2.1.3 Ehrenfest Theorem

When we derive Newton equation from quantum mechanics, it is generally called "classical limit" and in this case, we take the limit of $\hbar \to 0$.

Here, we explain the Ehrenfest theorem in which Newton equation can be obtained from Schrödinger equation by making the expectation values of $\boldsymbol{r}$ and $\boldsymbol{p} = -i\hbar\boldsymbol{\nabla}$. But before deriving the theorem, we should explain what is the expectation value in quantum mechanics.

• **Expectation Value :** An arbitrary operator $\hat{\mathcal{O}}$ which is a function of $\boldsymbol{r}$ and $\boldsymbol{p}$ can be taken over integrations by the wave functions as

$$\langle \Psi | \hat{\mathcal{O}} | \Psi \rangle \equiv \int \Psi^\dagger(\boldsymbol{r}) \hat{\mathcal{O}} \Psi(\boldsymbol{r}) d^3r \tag{2.4}$$

which is called "the expectation value of operator $\hat{\mathcal{O}}$". The representation of bra-ket is simply for convenience, and there is no special physical meaning.

• **Time Development of Schrödinger Equation :** Now we should calculate the time development of the expectation value of operator $\hat{\mathcal{O}}$. Since the Schrödinger equation is written as

$$i\frac{\partial \Psi(\boldsymbol{r},t)}{\partial t} = \hat{H}\Psi(\boldsymbol{r},t), \qquad -i\frac{\partial \Psi^\dagger(\boldsymbol{r},t)}{\partial t} = \hat{H}\Psi^\dagger(\boldsymbol{r},t), \tag{2.5}$$

the time differentiation of expectation value $\langle \Psi | \hat{\mathcal{O}} | \Psi \rangle$ becomes

$$i\frac{d}{dt}\langle \Psi | \hat{\mathcal{O}} | \Psi \rangle = \langle \Psi | \hat{\mathcal{O}} H | \Psi \rangle - \langle \Psi | H \hat{\mathcal{O}} | \Psi \rangle = \langle \Psi | [\hat{\mathcal{O}}, \hat{H}] | \Psi \rangle \tag{2.6}$$

where $[A, B] \equiv AB - BA$ is introduced.

• **Newton Equation :** Now as a simplest example, we take $\hat{H}$ as

$$\hat{H} = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + U(r) \tag{2.7}$$

which is one particle state. Let us take the $\hat{\mathcal{O}} = \boldsymbol{r}$ case. From eq.(2.6), we find

$$\frac{d}{dt}\langle \Psi | \boldsymbol{r} | \Psi \rangle = \frac{1}{m}\langle \Psi | \hat{\boldsymbol{p}} | \Psi \rangle, \qquad (\text{with } \hat{\boldsymbol{p}} \equiv -i\hbar\boldsymbol{\nabla}) \ . \tag{2.8}$$

In addition, we calculate the same procedure for the $\hat{\mathcal{O}} = \hat{\boldsymbol{p}}$ case, and we obtain

$$\frac{d}{dt}\langle \Psi | \hat{\boldsymbol{p}} | \Psi \rangle = -\langle \Psi | \boldsymbol{\nabla} U | \Psi \rangle. \tag{2.9}$$

To see more clearly, we define the following classical quantities

$$\boldsymbol{r} \equiv \langle \Psi | \boldsymbol{r} | \Psi \rangle, \quad \boldsymbol{p} \equiv \langle \Psi | \hat{\boldsymbol{p}} | \Psi \rangle, \quad \boldsymbol{\nabla} U(r) \equiv \langle \Psi | \boldsymbol{\nabla} U | \Psi \rangle \tag{2.10}$$

and then we have

$$m \frac{d^2 \boldsymbol{r}}{dt^2} = -\boldsymbol{\nabla} U(r) \tag{2.11}$$

which is just the Newton equation for the potential $U(r)$.

• **Potential and Force :** It should be very important to realize that the Newton equation is derived from the Schrödinger equation. In this case, we notice that there should not appear any concept of force in quantum mechanics, and everything is described in terms of potential $U(r)$. Therefore, in classical mechanics, we should take the force as the differential of potential. Here a question may arise as to why many types of potential like harmonic oscillator force may appear in classical mechanics. This is easy to answer, and if we want to reduce the many body problem to one body potential by any means, then we find a complicated potential which can be reduced to a harmonic oscillator potential in the case of small vibrations.

2.1.4 Fermion and Boson

Electron belongs to a fermion while photon is a boson. Fermion has a half integer spin $\frac{1}{2}$ and obeys the Pauli principle. As stable fermions, there exist nucleons (proton and neutron), electron and neutrino. It should be noted that neutron is stable only when it is bound in nucleus. On the other hand, photon is the only stable boson. However, since there is no rest system for photon, it is not so easy to discuss and understand the statistical behavior of photons. As an unstable boson, there are weak vector bosons ($W^{\pm}$, Z^0). But, again since they have only a very short life time, it is almost impossible to clarify their statistical behavior.

• **Spin or Polarization Vector of Vector Bosons :** The weak boson is a massive vector boson whose spin is 1. This is just the same as photon. However, this spin is not an eigenvalue of the spin angular momentum. In addition, the state vector of photon and massive vector boson should not be an eigenfunction of the spin angular momentum operator. However, from the group theoretical consideration, we know that the polarization vector of photon can be expressed as a spin 1 object due to the representation of rotation group. The polarization vector can be treated as a rank one tensor.

• **Pauli Principle :** Fermion should obey the Pauli principle, which is the most important property of fermion. This means that one quantum state can be occupied only by one fermion, and this must hold rigorously. This principle can be derived from the anti-commutation relation of the field quantization of fermion in field theory. In this sense, this is not a principle any more, and it is a physical law.

• **Bose Condensation :** The terminology of bose condensation is well known, but this is only the theoretical claim. Indeed, there should not be any phenomena that correspond to the bose condensation. The reason is simple. There is no stable elementary particle of boson except photon, but photon has no rest system. In this respect, the bose nature of photon becomes important only when we derive the Planck distribution in black body radiation.

On the other hand, there is some claim that the complex object like atoms whose spin should be integer must obey the bose statistic. However, this claim has neither theoretical foundation nor experimental evidence, and the statistical behavior of complex objects must be carefully examined.

2.1.5 Free Electrons

In quantum mechanics, it should not be very easy to properly understand the quantum state of free electron. But at the same time, a free electron plays a very important role in physics, and therefore we should make a brief but interesting explanation of free electron waves.

The free electron state is not a bound state as clear from the definition, and therefore, this electron can be found in any place in the world. However, it is ridiculous to discuss a possibility that it may be found on the moon. Therefore, we should treat electrons which are localized somewhere around where we are working, and thus we take a picture that a free electron should be trapped in a box with its size of L.

• **Schrödinger Equation of Free Electron :** The Schrödinger equation of a free electron with the mass of m can be written

$$-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r}). \tag{2.12}$$

Here it should be noted that the energy eigenvalue E must be positive. Now we define the wave number k as $k = \sqrt{\frac{2mE}{\hbar^2}}$. In this case, eq.(2.12) becomes

$$(\boldsymbol{\nabla}^2 + k^2)\psi(\boldsymbol{r}) = 0. \tag{2.13}$$

This differential equation (2.13) can be easily solved, and its solution becomes

$$\psi(\boldsymbol{r}) = Ae^{i\boldsymbol{k}\cdot\boldsymbol{r}} + Be^{-i\boldsymbol{k}\cdot\boldsymbol{r}}. \tag{2.14}$$

In this way, the Schrödinger equation of a free electron is properly solved, but this is not sufficient in quantum mechanics. Since the Hamiltonian of free electron commutes with the momentum operator $\hat{\boldsymbol{p}} \equiv -i\hbar\boldsymbol{\nabla}$, we should require that the solution must be also the eigenstate of the momentum operator

• **Eigenstate of Momentum Operator :** A free electron must be an eigenstate of $\hat{\boldsymbol{p}} \equiv -i\hbar\boldsymbol{\nabla}$. Therefore, we find

$$\hat{\boldsymbol{p}}\psi(\boldsymbol{r}) = \hbar\boldsymbol{k}\psi(\boldsymbol{r}). \tag{2.15}$$

From this condition, we find $A = 0$ or $B = 0$. Here we take $B = 0$. In this case, eq.(2.14) becomes

$$\psi(\boldsymbol{r}) = Ae^{i\boldsymbol{k}\cdot\boldsymbol{r}}. \tag{2.16}$$

2.1.6 Periodic Boundary Condition

In order to obtain the energy eigenvalue, we should impose the boundary condition on the wave function. For free electrons, we put the electron into a box with its length of L, and impose the periodic boundary condition (PBC) on the wave function

$$\psi(x, y, z) = \psi(x + L, y + L, z + L). \tag{2.17}$$

We insert $\psi(\boldsymbol{r}) = Ae^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ into this equation. Thus, $\boldsymbol{k}$ is determined as

$$\boldsymbol{k} = \frac{2\pi}{L}\boldsymbol{n}, \quad (n_x = 0, \pm1, \pm2, \cdots,\ n_y = 0, \pm1, \pm2, \cdots,\ n_z = 0, \pm1, \pm2, \cdots).$$

• **Boundary Condition at Fixed End :** Waves that propagate in medium should have real wave functions, and usually the wave can be written in terms of sin or cos functions. In this case, we impose the boundary condition at the fixed end. The oscillation of medium has nothing to do with its probability of wave, and therefore, there is no problem that the wave function of the medium oscillation vanishes at the edge. On the other hand, the wave function of free electron is directly related to the probability of electron and therefore, the wave function should not vanish at any point of space. Indeed, the wave function of $e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ is always finite, and in addition, the absolute value of $e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ is unity. In this sense, the PBC is quite reasonable as the boundary condition in quantum mechanics.

• **Normalization Condition :** The free electron state $\psi(\boldsymbol{r})$ is confined in the box of volume $V = L^3$. Therefore, the wave function $\psi(\boldsymbol{r})$ can be written as

$$\psi(\boldsymbol{r}) = \frac{1}{\sqrt{V}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}, \quad \boldsymbol{k} = \frac{2\pi}{L}\boldsymbol{n} \ \ (\boldsymbol{n} : \text{ integer }).$$

Here it is easy to prove that this state vector is normalized according to

$$\int_V |\psi(\boldsymbol{r})|^2 d^3r = 1.$$

• **Energy Eigenvalues E_n of Free Electron State :** From these considerations, we see that the energy eigenvalues E_n of free electron state can be given as

$$E_n = \frac{\boldsymbol{k}^2\hbar^2}{2m} = \frac{(2\pi\hbar)^2}{2mL^2}\boldsymbol{n}^2 \qquad (\boldsymbol{n} \ : \ \text{integer}).$$

Here it should be noted that the box length L appears in the energy eigenvalue even though L should not be taken as a natural length. However, in reality, the box size L should be taken sufficiently larger than any length scale of electrons, such as $\frac{\hbar}{mc}$. Indeed, it is clear that any physical observables should not depend on L, and this should be always satisfied.

2.2 Electron and Electromagnetic Fields

If there exist some macroscopic number of electrons, then the properties of electrons should be determined from the electromagnetic law of physics. Therefore, their collective behaviors can be understood by the Maxwell equation.

2.2.1 Static Electricity

Electricity was known a few hundred B.C. in ancient Greek. If one rubs amber with blanket, then one finds some electricity induced, and this is known as the origin of electron (electron is amber in Greek).

• **Frictional Charging :** A certain substance (dielectrics) can be charged by the friction. Everybody may well have this type of experience that there is some shock due to the frictional electricity. Among substances, one is charged positively and the other negatively. This cause of charging is originated from the stripping of electrons from one substance away.

2.2.2 Coulomb Force

The force of static electricity is the Coulomb interaction, and this works between charged particles. The strength of the Coulomb force is proportional to the number of charged particles. Further, there are positive and negative charges, and electron has a negative charge while proton has a positive charge. For the same charged particles, the force between them is repulsive while it is attractive for the different charged particles.

2.2.3 Piezoelectric Effect

There is an interesting phenomenon which is called a piezoelectric effect. When one puts a mechanical pressure on some crystal substance (soft matter), then this substance makes electric polarizations which can induce electric displacements. These phenomena were discovered by Pierre Curie 100 years ago.

In these phenomena, the mechanical force on the crystal should break some of its symmetry and thus, the force can induce the polarization of the crystal. This induced polarization is directly related to the electromagnetic effects. Therefore, the piezoelectric effect is to connect the mechanical force to the electric interaction, and therefore, this effect is made for many kinds of applications. In particular, it is, by now, very popular that, by touching liquid crystal display, one can transform the hand signal into the electric signal.

2.2.4 Current and Battery

It is known in science history that the electric current was discovered by Galvani when he made the experiment on the convulsion of frog muscle. Indeed, if one prepares salt water between two metal plates, then there should appear some potential difference, and thus, electricity is induced.

• **Electric Current :** The cause of this phenomenon is clarified experimentally by Volta who showed that two different kinds of metal plates that sandwich papers with salt water can produce some electric currents due to electric potential differences.

• **Battery :** Even if we collect ample static electricity, this electricity is discharged immediately. On the other hand, the battery is completely different, and it is very useful since the battery can be used at any time we want. Therefore, the battery has obtained an enormous value of use since then.

2.2.5 Maxwell Equation

In order to understand electromagnetisms, we should have to remember Maxwell equation. This is the most reliable theoretical framework which is well examined by all kinds of experiments. Nevertheless, to the readers of this short lectures, it may not be very easy to make use of the Maxwell equation. Maxwell equation is composed of the following four equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \quad \text{(Gauss Law)} \tag{2.18}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \quad \text{(No Magnetic Monople)} \tag{2.19}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \quad \text{(Faraday Law)} \tag{2.20}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{j} \quad \text{(Ampère-Maxwell Law)} \tag{2.21}$$

where ρ and $\boldsymbol{j}$ denote the charge and current densities which are related to the electron motions in the materials. In fact, the charge and current densities are most complicated due to the many body interactions in quantum mechanics.

• **Unknown Functions :** In the Maxwell equation, there are two unknown vector functions which are electric field of $\boldsymbol{E}$ and magnetic field of $\boldsymbol{B}$, and therefore, total unknown functions amount to 6. Indeed the Maxwell equation has 6 independent equations. Gauss law (1), no magnetic monopole (1), Faraday law (2) and Ampère-Maxwell law (2). At a glance, it may well be that the Ampère-Maxwell law should have three independent equations. However, due to the continuity equation

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0$$

the number of independent equations become 2. In fact, the second term in eq.(2.21) is introduced such that the continuity equation should hold.

• **Definition of Field :** Here we should explain what is the meaning of "fields" $\boldsymbol{E}$ or $\boldsymbol{B}$. This is quite simple, and both $\boldsymbol{E}$ and $\boldsymbol{B}$ depend on time and space. There is nothing more than that, and mainly this is in contrast to classical mechanics where one should only be interested in the time development of particle motion. As we note above, the charge and current densities are constructed from quantum mechanical wave function $\psi(t, \boldsymbol{r})$ which is also a field. In this respect, quantum mechanics and electromagnetisms both belong to the field theory framework. In addition, it should be noted that the gravity theory is also described in terms of field theory terminology [4, 5].

2.2.6 Electron and Electromagnetic Interactions

The interaction Hamiltonian between electron and electromagnetic field can be written as

$$H' = -e \int \boldsymbol{j}(\boldsymbol{r}) \cdot \boldsymbol{A}(\boldsymbol{r}) d^3 r \tag{2.22}$$

where $\boldsymbol{j}(\boldsymbol{r})$ denotes the current density of electron. This expression is written in terms of field theory language. However, it may be better to use the quantum mechanical expression in terms of non-relativistic Hamiltonian, and thus we briefly explain how we can calculate physical observables.

• **Non-relativistic Electron and Electromagnetic Interactions :** For a non-relativistic electron, the electromagnetic Hamiltonian can be written as

$$H = \frac{1}{2m} \left[\boldsymbol{\sigma} \cdot \left(\hat{\boldsymbol{p}} - \frac{e}{c} \boldsymbol{A} \right) \right]^2 - \frac{Ze^2}{r} \tag{2.23}$$

where $\boldsymbol{\sigma}$ denotes Pauli matrix and this is 2 by 2 hermite matrix which is written as

$$\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z), \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Here noting that $\hat{\boldsymbol{p}}$ is an operator ($\hat{\boldsymbol{p}} = -i\hbar \boldsymbol{\nabla}$) and also making use of the following mathematical formula

$$\left[\boldsymbol{\sigma} \cdot \left(\hat{\boldsymbol{p}} - \frac{e}{c} \boldsymbol{A} \right) \right]^2 = \left(\hat{\boldsymbol{p}} - \frac{e}{c} \boldsymbol{A} \right)^2 - \frac{ie}{c} \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}} \times \boldsymbol{A}$$

we obtain

$$H = \frac{1}{2m}\hat{\boldsymbol{p}}^2 - \frac{Ze^2}{r} - \frac{e}{2mc}(\hat{\boldsymbol{L}} + \hbar\boldsymbol{\sigma}) \cdot \boldsymbol{B}. \tag{2.24}$$

Here, $\hat{\boldsymbol{L}}$ denotes the angular momentum ($\hat{\boldsymbol{L}} = \boldsymbol{r} \times \hat{\boldsymbol{p}}$) and the magnetic field $\boldsymbol{B}$ is written as $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$. Also the uniform magnetic field is assumed. In this case, the third term in eq.(2.24) corresponds to the Zeeman effect.

• **Gradient Magnetic Field and MRI :** Proton in hydrogen atom under the strong magnetic field has the Zeeman splitting, and it emits a photon with its energy corresponding to the Zeeman splitting. However, the wave length of this radiation is quite long, and it is impossible to identify the place of proton which emits the radiation.

However, if we make use of the gradient magnetic field method, then we can identify the place of proton that emits the corresponding photon. This method is developed to the MRI (Magnetic Resonance Imaging). Here we briefly explain the principle of MRI.

• **Zeeman Effect Energy and Radiation :** The spin of proton is $\frac{1}{2}$ and the splitting energy of two spin states becomes $\boxed{\Delta E = \frac{e\hbar B}{Mc}}$. A photon corresponding to this energy is emitted.

• **Gradient Magnetic Field :** The idea of the gradient magnetic field method is introduced by Lauterbur. This method is to determine the place of proton that emits a photon with the Zeeman splitting energy. The frequency corresponding to the Zeeman splitting energy of proton is $\omega = \frac{eB}{Mc}$. Here we assume that the external magnetic field B should have special coordinate dependence. For simplicity, we consider one dimensional case and assume that B should have the following coordinate dependence in the range of $0 < x < a$

$$B = B_0 \, x \qquad (0 < x < a). \tag{2.25}$$

In this case, ω and x should have a one to one correspondence, and this can be understood from the equation $x = \frac{\omega Mc}{eB_0}$. Therefore, if one can measure the ω on the photon from proton, then one can determine the place of the proton x that emits the photon. This explanation is done in one dimension, but this can be extended to three dimensions. This leads to the observation in which way H_2O molecules should be distributed in human body, and this is the basic mechanism of the MRI method.

2.2.7 Lorentz Force

Lorentz force $\boldsymbol{F}$ is the force acting on electron if there is a magnetic field of $\boldsymbol{B}$ that is produced by some other charged particles. In this case, the Lorentz force can be written as

$$\boldsymbol{F} = e\boldsymbol{v} \times \boldsymbol{B} \tag{2.26}$$

where $\boldsymbol{v}$ denotes the velocity of electron. This force is originated from the interaction of H' in eq.(2.22).

2.2.8 Motor

The electric current should receive the Lorentz force from magnetic field since the current is related to the motion of electrons, and this is the basic mechanism of motor.

• **Commutator and Motor :** Electrons in the coil current under the magnetic field $\boldsymbol{B}$ should receive the Lorentz force $\boldsymbol{F} = e\boldsymbol{v} \times \boldsymbol{B}$. Using this force, motor is designed such that the coil can rotate continuously, and in this case, the current of the coil should be changed according to the rotation of the coil which is controlled by the commutator.

2.3 Current: Conductor and Semi-Conductor

The basic mechanism of current is described as the phenomena in which electrons weakly bound jump successively from one atom to the neighboring atoms. Therefore, from A-point to B-point of macroscopic distance, electrons altogether move A to B immediately. Indeed the time of information current from A to B is just the time of one electron jumping between two nearby atoms, and this is, in fact, instantaneous.

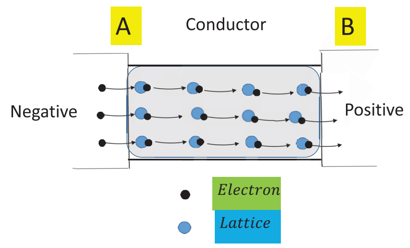

Figure 2.2: Picture of Current

2.3.1 Conductor

Conductors are defined as the substances in which many electrons can jump from one atom to the other when there is a electric field, and copper or these metals belong to conductors.

• **Ohm's Law :** Ohm's law is known when the current density $\boldsymbol{j}$ and electric field $\boldsymbol{E}$ are related to each other as

$$\boldsymbol{j} = \kappa \boldsymbol{E} \tag{2.27}$$

which is a phenomenological equation. Here, κ is called conductivity and it is related to the current flow as to how easily electrons can move. Therefore, conductivity is proportional to the number of electrons that can transfer from one atom to the other. But at the same time, it should depend on the probability of exciting the atoms when electrons jump at the nearby atoms. But these calculations of electron transfer is quite difficult and therefore, no microscopic calculation has been done yet until now. Also, Ohm's law violates the time reversal invariance, and thus it cannot be considered as the fundamental equation. However, it is indeed true that this should hold quite well for some substances of conductors.

2.3.2 Semi-Conductor

The semi-conductor is often used in the electronics of daily life. Especially nowadays, the semi-conductor diode is used as current rectifiers, in place of vacuum diode. As a substance of semi-conductor, germanium may be most well-known.

• **Water is Semi-Conductor?:** What about water? Is that semi-conductor? Below we see the electric resistivity of pure water and sea water.

Pure Water : $\sigma \sim 2.5 \times 10^5 \quad \Omega \cdot \mathrm{m}$
Seawater : $\sigma \sim 2.0 \times 10^{-1} \quad \Omega \cdot \mathrm{m}$

Therefore, we see that seawater can be considered as a semi-conductor. The difference between seawater and pure water must come from the fact that seawater should contain many ions and thus the number of movable electrons must be much larger than that of pure water.

2.3.3 Magnetization and Magnets

In conductor, some quasi-free electrons under the magnetic field should make circular motions which should induce the magnetization.

• **Magnetization :** The circular motion corresponds to the magnetic dipole moment, and thus it creates the internal magnetic field. The magnetization is the sum of these internal magnetic fields, and these fields are made so as to cancel out the external magnetic field. This is because nature always chooses the physical state in which the total energy of magnetic fields in the substance should be minimized.

• **Magnets :** In most cases, the internal magnetization should vanish when the external magnetic field is switched off. However, in some of the substances, the internal magnetic field does not disappear, and this is magnet. But in nature there is no magnetic monopole, and therefore, the internal magnetization of magnets should be made by the magnetic dipole moments. In normal cases, after the switch-off of the external field, the directions of the magnetic dipole moments should start fluctuating randomly and thus averaging over the magnetic dipole moments, one should find no magnetic field left. However, due to some unknown reasons, the directions of $\boldsymbol{m}$ should stay in the same way for almost infinite time of period. In the magnets, the spin magnetic moments ($\boldsymbol{m} \simeq \frac{e}{m}\boldsymbol{s}$) should be more important than those of the circular motion of electrons. In this sense, the spin directions in the outer shell of atoms may align under the external magnetic field, and once aligned, the spin in the magnets should stay as it is, though this phenomena of the spin alignment is not understood theoretically at all. This is one of the most important problems which should be solved in solid state physics.

2.4 Summary of Electron Physics

Most of the physical phenomena in daily life can be understood in terms of electron dynamics.

2.4.1 What is Charge?

What is charge? This question is very important since most physicists do not understand it properly. If you ask this question as to what is the physical meaning of charge, then mostly you may find an answer that the charge must be "e". However, this answer is, of course, incorrect. The charge is a quantum number that belongs to a particle.

• **Quantum Number of Fermion Charge :** For example, electron has "-1" charge as a quantum number while proton has "$+1$" charge. These are fermions, and the quantum number of elementary fermions should not be zero if these fermions should have electromagnetic interactions. For example, neutrinos cannot have any electromagnetic interactions since their charge is zero. On the other hand, neutron should have electromagnetic interactions even though its quantum number is zero. This is connected to the fact that neutron is composed of three quarks which have finite charges.

• **Quantum Number of Boson Charge :** As an elementary boson, there exist only vector bosons in nature, and charged vector bosons are $W^{\pm}$. However, they do not have any electromagnetic interactions, apart from a possibility that they may have some electromagnetic interactions in the higher order process of weak interactions, but this is not confirmed yet.

2.4.2 What is Current?

What is electric current? In this lecture, we make a brief explanation of the current in terms of jumping electrons from one atom to the other, and this jump must be simultaneous. In this respect, electric resistance can be understood as the interactions of electrons with nearby atoms which may get excited through the interactions. However, this type of calculations are so difficult that no concrete evaluation has been done yet, even approximately.

• **Superconductor :** Therefore, it is practically impossible to understand the physics of superconductor in terms of any theoretical models. Up to now, the BCS theory is known as a candidate of explaining the superconductor phenomena. However, the BCS theory is just to calculate some effects of the interactions, instead of evaluating the scattering problems.

2.5 Homework Problems

Answer the following questions with your own words.

2.1 The wave function $\psi(\boldsymbol{r})$ is a complex function. Therefore, $\psi(\boldsymbol{r})$ itself is not observable, and thus we consider $|\psi(\boldsymbol{r})|^2$. This corresponds to the probability. Why is it?

Hint : $\int |\psi(\boldsymbol{r})|^2 d^3r = 1$

2.2 By making some approximations, we can obtain classical mechanics. What should happen to the degree of freedom for both mechanics?

2.3 The free wave function of $\psi(\boldsymbol{r})$ is written as
$\psi(\boldsymbol{r}) = \frac{1}{\sqrt{V}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$, $\left(\boldsymbol{k} = \frac{2\pi}{L}\boldsymbol{n}, \quad \boldsymbol{n}; \text{ integer}, \quad V = L^3\right)$
Prove that this is normalized to unity.

2.4 In the Ehrenfest theorem, we calculate the time development of $\langle \boldsymbol{r} \rangle$ in order to derive the Newton equation in classical mechanics. The expectation value of the coordinate corresponds to the classical mechanics, and why is the expectation value so important?

2.5 In quantum mechanics, Hamiltonian H plays an important role. On the other hand, Hamiltonian in classical mechanics is used when the Hamilton equations are derived, $\dot{\boldsymbol{r}} = \frac{\partial H}{\partial \boldsymbol{p}}, \quad \dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{r}}$. From the Hamilton equation, we obtain $\dot{\boldsymbol{r}} = \frac{\partial H}{\partial \boldsymbol{p}}, \quad \dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{r}}$. In this case, what is the physical meaning of Hamiltonian?

2.6 The reference point of the potential in classical mechanics has an ambiguity how to choose it. What should be a reason for it? In quantum mechanics, how can you determine the reference point of the potential?

2.7 There is always Pauli principle for fermions. In case the mass of fermion is zero, what should we do?

(For massless fermion, we cannot find a rest frame.)

2.8 The free wave in quantum mechanics is one body problem, and thus, it is simple. However, classical wave such as sound wave is very difficult to treat. What should be a reason of this difficulty?

2.9 If operators A and B commute with each other $[A, B] = 0$, then they have the same eigenstate for them. Prove it.

2.10 Thunder is a phenomenon in which big currents flow between thunder clouds and the earth or between thunder clouds. In this case, why does lightening occur? What should be the physical reason of thunderclap?

2.11 Prove that Maxwell equation has 6 independent equations.

2.12 Make your own picture how MRI(Magnetic Resonance Imaging) works by measuring proton distributions.

2.13 The communication time between mobile phone and usual telephone is just the same. Why is it so?

2.14 Ohm's law $\boldsymbol{j} = \kappa \boldsymbol{E}$ violates the time reversal invariance. Prove it.

2.15 The electric current is described as the jump of electrons from one atom to the other at the same time. In this case, what should be a physical mechanism of electric resistance?

2.16 Explain the mechanism of motor by drawing picture.

2.17 In some substances under the magnetic field, there appear the circular currents which make internal magnetizations. This internal magnetization is always to cancel out the external magnetization. Why is it so?

2.18 Charge is the quantum number of a particle. Weak vector bosons of $W^{\pm}$ has a charge but they do not react on the electromagnetic force. Why is it so?

Chapter 3. Physics of Electromagnetic Waves

Electromagnetic wave is a massless particle and therefore, its property should be characterized by its wave length. Electromagnetic wave is called photon which has two degrees of freedom, though its spin is 1.

In nature, a macroscale number of photons with various wave lengths should be involved, and therefore, this corresponds to light. In this case, light behaves almost like waves. But wave cannot propagate in vacuum, and in this respect, light is not a wave since the wave is defined as an object which should propagate in medium.

Electromagnetic waves are called in many different ways, depending on the wave length. Electromagnetic wave with the long wave length is called radio wave while, if short, it is called visible light or infra-red or ultra-violet light and so on. If it is even shorter, then it becomes X-ray or γ ray.

3.1 Electromagnetic Wave

Light is a most common physical quantity in daily life, but at the same time, it is most important to us. Light is named as radio wave, infra-red ray, background radiation and $\gamma-$ray, etc. But apart from the wave length, they are just the same light, that is, electromagnetic wave or photon.

Wave Length of Light and Its Property

The numbers in this table are just for a guide, and they are not necessarily accurate. Also Lamb shift is the energy difference between $2s_{\frac{1}{2}}$ and $2p_{\frac{1}{2}}$ in hydrogen atom, and it shows a corresponding wave length. The wave length of microwave shows the one that is used in microwave oven. The light with wave lengths larger than 1 cm is called radio wave.

E.M Wave	**Wave Length** $[\lambda]$	**Energy** (eV)	**Frequency**
MF	$\sim 0.4 \times 10^5$ cm	$\sim 0.4 \times 10^{-8}$	~ 1 MHz
VHF	$\sim 0.4 \times 10^3$ cm	$\sim 0.4 \times 10^{-6}$	
UHF	~ 40 cm	$\sim 0.4 \times 10^{-5}$	~ 1 GHz
Lamb Shifts	40 cm	0.44×10^{-5}	1 GHz
Microwave	$15 \sim 45$ cm	$\sim 10^{-5}$	$0.9 \sim 2.5$ GHz
Background R.	0.8 cm	2.5×10^{-4}	50 GHz
Infra-red	$\geq 10^{-3}$ cm	≤ 0.1	$\leq 2 \times 10^{13}$ Hz
Red	0.8×10^{-4} cm	1.6	4×10^{14} Hz
Ultra-Violet	0.4×10^{-4} cm	3.3	8×10^{14} Hz
Ultra-Violet	$\leq 10^{-5}$ cm	≥ 10	$\geq 2 \times 10^{15}$ Hz
X-ray	~ 1 Å	$\sim 10^3$	
γ Ray	$\leq 10^3$ fm	$\geq 10^6$	

3.1.1 Theory of Photon

In order to understand photon, we should go beyond the Maxwell equation, and therefore, it is not so easy to explain. In most of the textbooks of electromagnetism, they claim that electromagnetic wave can be understood from the Maxwell equation, but this is not correct as we show below. First we rewrite the Maxwell equation in terms of vector potential without any currents

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2\right)\boldsymbol{A} = 0. \tag{3.1}$$

Indeed, this equation is satisfied by free photon. However, this equation only suggests some existence of photon, but it has nothing to do with the creation or annihilation of photon. To understand the creation or annihilation of photon, one has to consider the field quantization. In fact, photon can become a physically meaningful object only if we carry out the field quantization which can treat the creation or annihilation of photon as we see below [3, 4, 5].

3.1.2 Photon

Photon is a massless particle which is originated from the field quantization of vector potential. When making the field quantization, we first expand the vector potential $\boldsymbol{A}$ in terms of free particle states as

$$\boldsymbol{A}(x) = \sum_{\boldsymbol{k}} \sum_{\lambda=1}^{2} \frac{1}{\sqrt{2V\omega_{\boldsymbol{k}}}} \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} \left[c^{\dagger}_{\boldsymbol{k},\lambda} e^{-i\omega_{\boldsymbol{k}}t + i\boldsymbol{k}\cdot\boldsymbol{r}} + c_{\boldsymbol{k},\lambda} e^{i\omega_{\boldsymbol{k}}t - i\boldsymbol{k}\cdot\boldsymbol{r}} \right] \tag{3.2}$$

where $\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}$ denotes the polarization vector. Now in order to describe the state vector of photon, we should make a field quantization.

• **Field Quantization :** The physics of field quantization is to consider the coefficients $c^{\dagger}_{\boldsymbol{k},\lambda}$ and $c_{\boldsymbol{k},\lambda}$ as operators that should satisfy the following commutation relations

$$[c_{\boldsymbol{k},\lambda},\ c^{\dagger}_{\boldsymbol{k}',\lambda'}] = \delta_{\boldsymbol{k},\boldsymbol{k}'}\delta_{\lambda,\lambda'}, \qquad [c_{\boldsymbol{k},\lambda},\ c_{\boldsymbol{k}',\lambda'}] = 0 \tag{3.3}$$

where $c^{\dagger}_{\boldsymbol{k},\lambda}$ abd $c_{\boldsymbol{k},\lambda}$ are called creation and annihilation operators, respectively. In this respect, photon has nothing to do with electric field $\boldsymbol{E}$ nor magnetic field $\boldsymbol{B}$, and photon is a massless particle with some wave behaviors.

• **Energy, Momentum and Wave Length of Photon :** The energy of photon $E_{\boldsymbol{k}}$, its angular frequency $\omega_{\boldsymbol{k}}$, the momentum $\boldsymbol{p}$ and the wave number $\boldsymbol{k}$ should be related to each other by

$$E_{\boldsymbol{k}} = \hbar\omega_{\boldsymbol{k}}, \quad \boldsymbol{p} = \hbar\boldsymbol{k}. \tag{3.4}$$

Further, the wave number and the wave length λ are related by $\boxed{\lambda = \frac{2\pi}{k}}$.

3.1.3 Interaction of Electron with Electromagnetic Field

In most of the textbooks of electromagnetism, it is written that photon is related to the electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$. But as explained above, this statement is incorrect. Photon is a massless particle derived from quantized vector potential $\boldsymbol{A}$. Therefore, it has nothing to do with $\boldsymbol{E}$ and $\boldsymbol{B}$, and indeed there is no reason to make any connections between photon and electric or magnetic fields of $\boldsymbol{E}$ and $\boldsymbol{B}$. The interaction Hamiltonian is written as

$$H_I = -e \int \boldsymbol{j} \cdot \boldsymbol{A} \, d^3 r.$$

From the shape of H_I, one sees that e corresponds to the interaction strength and it is called "coupling constant". It should be also important to note that the creation and annihilation of photon should take place only by this interaction since the vector potential is quantized only in eq.(3.2).

3.1.4 State Vector of Photon

The vector potential $\boldsymbol{A}$ is a real function since electric and magnetic fields are both real functions. However, the state vector of photon should be a complex function since it should propagate as a free wave. Indeed the state vector of photon $\boldsymbol{\Psi}_{\boldsymbol{k\lambda}}$ is defined as

$$\boldsymbol{\Psi}_{\boldsymbol{k\lambda}} \equiv \langle \boldsymbol{k}, \lambda | \boldsymbol{A}(x) | 0 \rangle = \frac{\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}}{\sqrt{2\omega_k V}} e^{i\omega t - i\boldsymbol{k}\cdot\boldsymbol{r}} \tag{3.5}$$

which is a complex function. The polarization vectors $\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}$ should have the following constraints

$$\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} \cdot \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda'} = \delta_{\lambda,\lambda'}, \qquad \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} \cdot \boldsymbol{k} = 0. \tag{3.6}$$

The first equation shows the orthogonality condition while the second one indicates that the polarization vector must be orthogonal to the momentum of photon. Here, the most important but difficult point is that there is no rest frame for photon, and thus it is practically impossible to make any physical picture of the polarization vector.

• **Spin of Photon :** In reality, the polarization vector plays an important role only at the instant when photon interacts with electron. Therefore, the description of the polarization vector should be physically meaningful at the rest frame of electron.

The spin of photon is 1, but its degree of freedom is 2, instead of three. At the instant of the interaction the conservation law of spin holds. However, the polarization vector does not satisfy the Lie algebra, and thus it is somewhat different from the spin itself. Since the polarization vector is a rank one tensor in rotation group, one can treat it almost in the same way as the spin 1 state.

3.1.5 Oscillator Mechanism of Electromagnetic Wave

Here we should make a brief explanation of oscillator mechanism of electromagnetic wave. The creation of photon is due to the electromagnetic interaction as written

$$H_I = -\int \boldsymbol{j} \cdot \boldsymbol{A} \, d^3r \tag{3.7}$$

where e is included in $\boldsymbol{j}$. Now the time differential of this interaction becomes

$$W \equiv \frac{dH_I}{dt} = -\int \left[\frac{\partial \boldsymbol{j}}{\partial t} \cdot \boldsymbol{A} + \boldsymbol{j} \cdot \frac{\partial \boldsymbol{A}}{\partial t} \right] d^3r \tag{3.8}$$

where electric field is written as

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t}. \tag{3.9}$$

Therefore, W becomes

$$W = -\int \frac{\partial \boldsymbol{j}}{\partial t} \cdot \boldsymbol{A} \, d^3r + \int \boldsymbol{j} \cdot \boldsymbol{E} \, d^3r \tag{3.10}$$

The second term in eq.(3.10) is expressed in terms of the electric field, and thus, it has nothing to do with the electromagnetic wave. By denoting the first term of eq.(3.10) by W_1, we should now evaluate the following quantity

$$W_1 \equiv -\int \frac{\partial \boldsymbol{j}}{\partial t} \cdot \boldsymbol{A} \, d^3r. \tag{3.11}$$

• **Hamiltonian of Zeeman Effect :** Here we employ the non-relativistic quantum mechanics. The interaction Hamiltonian is due to the Zeeman effect term which is written as

$$H = -\frac{e}{2m_e} \boldsymbol{\sigma} \cdot \boldsymbol{B}_0 \tag{3.12}$$

where we assume that the external magnetic field $\boldsymbol{B}_0$ should depend on the coordinate $\boldsymbol{r}$. Here we take the direction of $\boldsymbol{B}_0$ to be $z-$direction, and thus it is written as $\boldsymbol{B}_0 = B_0(\boldsymbol{r})\boldsymbol{e}_z$. In this case, the current $\boldsymbol{j}$ becomes

$$\boldsymbol{j}(t, \boldsymbol{r}) = \frac{e}{m_e} \psi^\dagger(t, \boldsymbol{r}) \, \hat{\boldsymbol{p}} \, \psi(t, \boldsymbol{r}) \tag{3.13}$$

where $\hat{\boldsymbol{p}} = -i\boldsymbol{\nabla}$. Therefore, we obtain

$$\frac{\partial \boldsymbol{j}(t,\boldsymbol{r})}{\partial t} = \frac{e}{m_e}\left[\frac{\partial \psi^\dagger}{\partial t}\hat{\boldsymbol{p}}\psi + \psi^\dagger \hat{\boldsymbol{p}}\frac{\partial \psi}{\partial t}\right] = -\frac{e^2}{2m_e^2}\boldsymbol{\nabla} B_0(\boldsymbol{r}). \tag{3.14}$$

In this way, the time differential of energy flow of electromagnetic wave becomes

$$W_1 = \int \frac{e^2}{2m_e^2}(\boldsymbol{\nabla} B_0(\boldsymbol{r}))\cdot \boldsymbol{A}\, d^3r. \tag{3.15}$$

The electromagnetic wave is produced from the $\boldsymbol{A}$ in the above equation.

3.2 Light and Wave

The problem of wave propagation is very difficult to physically understand even for experts in this field. Here, we make a brief explanation of sound wave by comparing with light. There is an essential difference between sound wave and light. Light is a particle whose state vector ψ (coordinate dependence only) can be written as

$$\psi \simeq A e^{i\omega t - i\boldsymbol{k}\cdot\boldsymbol{r}} \tag{3.16}$$

while the wave function of sound wave ϕ becomes

$$\phi \simeq A \sin(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}) \tag{3.17}$$

which is a real function. This realness of the wave function comes from the fact that the sound wave should propagate in medium as the oscillation. On the other hand, the absolute magnitude of the state vector of photon is $|e^{i\omega t - i\boldsymbol{k}\cdot\boldsymbol{r}}| = 1$ and this is related to the fact that the probability of finding photon must be always finite. On the other hand, the oscillation of medium can vanish at some point of space.

As stated in the previous section, the state vector of photon $\boldsymbol{\Psi}_{\boldsymbol{k}\boldsymbol{\lambda}}$ is written

$$\boldsymbol{\Psi}_{\boldsymbol{k}\boldsymbol{\lambda}} = \frac{\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}}{\sqrt{2\omega_k V}} e^{i\omega t - i\boldsymbol{k}\cdot\boldsymbol{r}}. \tag{3.18}$$

This shows that photon is a free particle, and therefore, the treatment is just simple. From the shape of wave function, one sees that it should have some properties of wave such as interferences or diffractions.

3.2.1 Doppler Effect of Light

Here we make a brief explanation of Doppler effect of light. These phenomena should be quite similar to the Doppler effect of sound. If the sound source is approaching to you, then you find that the frequency of sound becomes higher, and this is just the same as the Doppler effect of light. But the Doppler effect of light is simpler than the sound case because the light is a particle and therefore, this is a two-body problem while sound is propagating in medium, and thus the Doppler effect of sound becomes a three-body problem, the source of sound, observer and medium (atmosphere or sea water).

• **Lorentz Transformation :** Here we write the Lorentz transformation which should hold for a particle with its energy E and momentum $\boldsymbol{p}$

$$p_x{}' = \gamma\left(p_x - \frac{vE}{c^2}\right), \quad E' = \gamma\left(E - vp_x\right), \quad p_y{}' = p_y, \quad p_z{}' = p_z \tag{3.19}$$

where E' and $\boldsymbol{p}'$ denote energy and momentum in the moving frame. Now we assume that a star is moving away from the earth with its velocity of v. If this star emits light with its wave length of λ, then the momentum of light becomes $p = \frac{\hbar c}{\lambda}$. In this case, the momentum we observe on the earth becomes

$$p' = \gamma\left(p - \frac{vE}{c^2}\right) = \gamma\left(p - \frac{vp}{c}\right) = \frac{p\left(1 - \frac{v}{c}\right)}{\sqrt{(1 - \frac{v^2}{c^2})}} = p\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}. \tag{3.20}$$

Therefore, the momentum of light becomes smaller, and if we write it in terms of wave length, then we find

$$\lambda' = \lambda\sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}. \tag{3.21}$$

The wave length of light becomes larger and thus, it is called "red shift". But this naming of the red shift has nothing to do with physics.

3.2.2 Sound Wave

In daily life, sound is most important. However, it is not so simple to understand its physics since sound propagates in medium as a density wave. This means that we have to describe the oscillation of medium density at every point of space, and this is practically impossible to treat its dynamics in an exact way. Also, sound propagates in three dimensions, and we see its difficulty.

Here we write the simplest version of the wave equation for sound, and this is an equation for the displacement of medium $u(\boldsymbol{r}, t)$

$$\left(\frac{\partial^2}{\partial t^2} - v^2 \boldsymbol{\nabla}^2\right) u(\boldsymbol{r}, t) = 0 \tag{3.22}$$

where $v \equiv \sqrt{\frac{P}{\rho}}$ corresponds to the velocity of sound. Here, ρ and P denote the density and pressure of atmosphere, and they are assumed to be a constant.

• **Ultra-Sound :** If one creates ultra-sound, then the expansion of this wave during the propagation is rather small, and therefore, it becomes similar to the beam. In this case, the ultra-sound is easier to handle. Therefore, there should be a wide range of applications of the ultra-sound such as ultrasonic fish finder or ultrasonic apparatus.

3.2.3 Doppler Effect of Sound

The Doppler effect of sound is not so easy to understand as compared with that of light. This is connected to the fact that the Doppler effect of sound involves sound, medium and observer, and thus it is a three body problem.

• **Doppler Effect : Observer (v_{ob}) Moves to Sound Source :** When the observer with the velocity of v_{ob} moves to sound source, then, this is just the same as the case of light since the sound source and medium are in the same system. Therefore, this can be just treated in terms of relativity (Galilei transformation). Denoting the velocity of sound by v, the frequency n' due to the Doppler effect becomes

$$n' = \frac{v + v_{ob}}{v} n. \tag{3.23}$$

• **Doppler Effect : Sound Source (v_{ss}) Moves to Observer :** On the other hand, when the sound source with the velocity of v_{ss} approaches to the observer, then the observer and medium are in the same system, and therefore, the observed wave length λ' should be shrinked and can be written as

$$\lambda' = \frac{v - v_{ss}}{v} \lambda. \tag{3.24}$$

3.3 Wave Length and Polarization Vector

The basic property of photon should be characterized by the wave number vector $\boldsymbol{k}$ and the polarization vector $\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}$.

• **Polarization Vector $\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}$:** The polarization vector $\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}$ depends on $\boldsymbol{k}$ and quantum number λ of photon. This polarization vector $\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}$ can be considered as a spin of photon which is 1. However, this vector does not satisfy the Lie algebra, and thus, it is not the same as the angular momentum. In addition, the degree of freedom is just two, and thus it is different from a usual spin.

• **State of Polarized Light :** What should be the polarized light? Photon has two degrees of freedom which correspond to the spin states. Polarized light is composed of photons with one spin state. In this case, the quantum number which specifies the polarization state is λ that appears in the polarization vector. It should be important to note that the state of photon can be determined from its wave length and the polarization state. White light, for example, is composed of photons mixed up with various wave lengths and two polarization states. Here we summarize properties of light.

• **Sunlight :** Sunlight contains various wave lengths and two polarization states equally mixed. The mixing of many wave lengths can be seen from the rainbow.

• **Laser :** Laser rays should all have the same wave length. But the polarization state of laser depends on how they are created.

• **Polarized Light :** In polarized light, wave length should be mixed up, but only one polarized state is realized. Polarized light can be obtained by taking light through the polarizing plate.

3.3.1 Blue Sky and Photon Scattering

In the electromagnetic interaction, we see that photon is either created or annihilated. However, photon can interact with electron elastically at the second order of the perturbation theory. This is known as the Compton scattering.

• **Rayleigh Scattering Cross Section :** The blue sky can be understood in terms of scattering between light and atmospheric molecule like N_2, and its cross section can be written as

$$\sigma_R \simeq \sigma_{Thom} \left(\frac{\lambda_0}{\lambda} \right)^4 \tag{3.25}$$

where λ_0 is a constant. This cross section was derived by Rayleigh with classical electrodynamics, but in quantum mechanics, it can also be calculated by the scattering process of the second order perturbation theory of the electromagnetic interaction H'

$$H' = -\frac{e}{m_e} \boldsymbol{A} \cdot \boldsymbol{p}. \tag{3.26}$$

Here, however, we do not discuss it any more since the calculation should involve some complicated procedures.

• **Why is Sky Blue? :** From the Rayleigh scattering cross section, one sees that photon with shorter wave length has a larger cross section. Thus, blue light is scattered more than other visible light. Therefore, one observes the scattered light in the sky, which is blue.

• **Thomson and Compton Scattering**

In the process of photon-electron interactions, there is no absorption of photon if electron is free. Instead, there is an elastic scattering between photon and electron, and this is known as the Compton scattering. The non-relativistic limit of the Compton scattering is called Thomson scattering and the cross section is written as

$$\sigma_{Thom} = \frac{8\pi}{3} r_0^2$$

with $r_0 = \frac{e^2}{m_e c^2}$, classical electron radius.

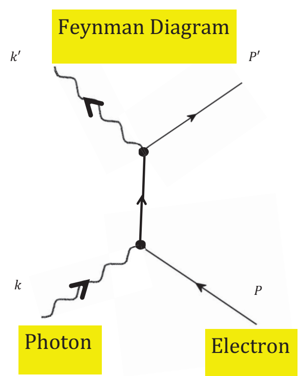

Figure 3.3: Compton Scattering

3.3.2 Black Body Radiation

Here we should explain the Planck formula that can describe the black body radiation. In this derivation, we assume that the energy of photon should be written as $E = h\nu$ with ν which is the frequency of photon. This means that the energy of photon is not continuous. Now we can derive the strength of the black body radiation I

$$I(\nu) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1} \tag{3.27}$$

where k denotes Boltzmann constant.

• **Stefan-Boltzmann Law:** Any substances with finite temperature should lose energy by radiations. For example, the earth is losing its energy from the surface by the black body radiation and this radiation energy can be obtained from the Stefan-Boltzmann law. This law can be derived by integrating eq.(3.27) over ν and we find

$$U = \sigma T^4, \quad \text{with} \quad \sigma \equiv \frac{8\pi k^4}{c^3 h^3} \int_0^\infty \frac{x^3 dx}{e^x - 1} \tag{3.28}$$

3.3.3 Solar Photoelectric Generation

The mechanism of power generation is mainly to make use of the electromagnetic induction. On the other hand, photoelectric effect is used for the solar photoelectric generation.

• **Photoelectric Effect :** Photoelectric effect is a phenomenon in which bound electron receives some energy from photon. In this case, electron which is bound in atoms with Z charge should absorb photon, and it should become free. The energy and momentum conservations can be written as

$$\boldsymbol{k} + \boldsymbol{p}_F = \boldsymbol{p}, \quad k + E_B = \frac{\boldsymbol{p}^2}{2m_e} \tag{3.29}$$

where $\boldsymbol{k}$ denotes the momentum of photon, $\boldsymbol{p}_F$ is the Fermi momentum of bound electron, and $\boldsymbol{p}$ denotes the momentum of electron after collision. The absorption probability of photon by the bound electron should be determined from the scattering cross section which is written as

$$\sigma_K \simeq \sigma_{Thom} \alpha^4 Z^5 2^{1.5} \left(\frac{m_e}{k}\right)^{3.5}. \tag{3.30}$$

3.4 Homework Problems

Answer the following questions with your own words.

3.1 The energy E and momentum $\boldsymbol{p}$ should have the following relation, $E = \sqrt{(mc^2)^2 + (\boldsymbol{p}c)^2}$. In this case, the velocity of this particle is defined as $\boldsymbol{v} = \frac{\boldsymbol{p}c^2}{E}$. Prove that the velocity of a massless particle becomes c.

3.2 Photon has no inertial system. People often discuss the circular polarization or linear polarization of photon. In which inertial system, do they discuss?

3.3 Photon is found always in the free state. That is, there is no bound state of photon. Why is it so?

3.4 Explain what you know about the background radiation.

3.5 The state vector of photon is $\boldsymbol{\Psi}_{\boldsymbol{k\lambda}} = \frac{\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}}{\sqrt{2\omega_k V}} e^{-ikx}$ which is a complex function. Then, what should be the probability of finding photon?

3.6 The state vector of sound wave is written in terms of a real function. Why is it necessary?

3.7 Photon has two degrees of freedom of spin. Suppose there are 100 photons here. What should be the number of photon with up spin?

3.8 The vector potential $\boldsymbol{A}$ is a real function. But the state vector of photon is complex. Why is it possible?

3.9 For the macroscale number (N) of electrons, we safely assume that the charge of this group of electrons becomes $Q = Ne$. Why is it so?

3.10 Usually a particle with spin 1 should have three degrees of freedom. But photon has only two degrees of freedom. Why is it so?

3.11 Suppose the sound velocity is v. The observer is moving away from the sound source with V. What should be the real speed of sound?

3.12 Explain for high school students why sky is blue.

3.13 Photoelectric effect cannot be described if we assume that light is wave. Why is it so?

3.14 The earth is radiating away the thermal energy in terms of black body radiation. Calculate the radiation energy of the earth using Stefan-Boltzmann law.

3.15 The average temperature of the earth is around $T \sim 300$ K. Estimate a typical wave length of the radiation.

3.16 The earth is losing its thermal energy only from the radiation on the surface. This radiation can be blocked when there should be clouds. Why is it so?

3.17 The radiation from the surface of the earth cannot be blocked unless there exist clouds. Some people (non-physicists) claim that radiations from the earth may be absorbed by CO_2 of the air, and this is the main reason of global warming. Why do they claim the absorption of radiation is important?

3.18 A recent study shows that the global warming rate due to the CO_2 absorption turns out to be less than 0.5 %. In this case, what do you think should be the main mechanism of the global warming?

Chapter 4. Gravity

Any massive object should fall on the earth because the gravitational force from the earth acts on this body. The force is attractive, and called " universal gravitation".

For a long time, physicists could not answer for the following questions as to why the gravity is always attractive and further, why the gravitational mass m is the same as the inertial mass m of the particle. In fact, it is only recent (about 10 years ago) that these fundamental questions are properly answered in terms of field theory terminology.

Gravity is a driving force to make stars, galaxies and universe. This universe is infinite, and infinite numbers of galaxies should exist. This "Mugen universe" has been existing indefinitely from the infinite past time, and should be the eternal existence in the infinite future.

4.1 Gravity

It is only recently that the theory of gravity is constructed in terms of field theory. Indeed it took quite a long time before the formulation of gravity is accomplished. What is the reason of this difficulty? Here, we should answer for this question as much as possible.

4.1.1 Why is Gravity Always Attractive ?

Gravity is essentially different from electromagnetic interactions. The electromagnetic interactions should have a repulsive force together with the attractive force. On the other hand, the gravitational force is always attractive and no repulsive interaction is found there. This indicates that the construction of the gravity theory should not start from the gauge theory.

How can we make attractive interactions, regardless charges or any other quantum numbers? In reality, this is rather simple. The interaction with the exchange of scalar field must be always attractive, which is a well-known fact. Therefore, a scalar field should be the best candidate. In addition, the gravitational potential should have the shape of $1/r$, and thus, this scalar field must be massless. This is the basic requirement in constructing the theory of gravity in field theory terminology.

4.1.2 Inertial Mass and Gravitational Mass

When a body falls onto the earth, the fall velocity does not depend on the mass of the falling body. In reality, due to the presence of atmosphere on the earth, the velocity depends on some properties of the body. But in vacuum state, the velocity of the falling body does not depend on the mass. This is simply because the gravitational mass and the inertial mass are just the same. Now the kinetic energy T of the body can be written as

$$T = \frac{1}{2} m \boldsymbol{v}^2 \tag{4.1}$$

where m and $\boldsymbol{v}$ denote the inertial mass and the velocity of the body, respectively. On the other hand, the gravitational potential $U(r)$ from the earth can be written as

$$U(r) = -\frac{G_0 m M}{r} \tag{4.2}$$

where G_0 is the gravitational constant. M and m denote the mass of the earth and the gravitational mass of the body, respectively.

Here the important point is that the masses appearing in the kinetic energy and gravitational potential are just the same as denoted by m. This looks like a natural result, but theoretically it is non-trivial, and this will be explained later in the new Gravity model. Now the equation of motion becomes

$$m\ddot{\boldsymbol{r}} = -\frac{G_0 mM}{r^3}\boldsymbol{r} \quad \Rightarrow \quad \ddot{\boldsymbol{r}} = -\frac{G_0 M}{r^3}\boldsymbol{r} \tag{4.3}$$

which shows that the motion does not depend on the mass m.

4.1.3 Connection with Electromagnetisms

The electromagnetism with the Maxwell equation is, at present, the most reliable theoretical frame work in modern physics. The basic point of the Maxwell equation is explained in chapter 2, and readers should refer to this chapter in order to understand it more in depth.

The electromagnetism is a field theory, and if we do not treat photon, then the electromagnetism is complete as a scheme in the theoretical frame work. If we want to include photon, then we should carry out the field quantization of vector potential, which is called quantum electrodynamics (QED). The quantization of fields induces creation and annihilation of particles and thus there appear some special phenomena such as the evaluation of $g-2$.

In contrast to QED, there is no observation of graviton, and indeed there is no possibility of observing gravitons since the strength of the gravity is smaller than the electromagnetic interaction by $\sim 10^{-35}$. Therefore, there is no necessity of quantization of gravitational field, which is a very important constraint when constructing the gravity theory.

4.1.4 Negative Effects of General Relativity on Gravity

Quite a few people may still believe that the general relativity must be a candidate for the theory of gravity. However, the general relativity is a theory for the coordinate system, and the unknown function is the metric tensor $g^{\mu\nu}$ which has nothing to do with the gravity. Einstein assumed that the metric tensor should be related to the gravitational potential. This is, of course, a ridiculous hypothesis since any unknown functions that should be obtained as solutions of the Einstein equation cannot be determined in advance. However, he postulated that the metric tensor should be related to the gravitational potential as $g^{00} \simeq 1 + 2\phi$ without any reasons.

In any case, the Einstein equation is constructed for the coordinate system of metric tensor $g^{\mu\nu}$, and he claimed that the theory should be related to the description of physical phenomena. However, there is a serious question as to why many people accepted his claim. Physics is to understand the motion of a particle which is described in the coordinate space. If one sets up any equation to change the scale of the coordinate system, then it has nothing to do with the motion of a particle, and this means that one would have no aim of understanding nature.

In this sense, Einstein did not understand the physical meaning of his equation, and this is the whole problem of the general relativity. It is really unfortunate that the vast amount of time and efforts should have been wasted due to this meaningless theory model.

• **General Relativity and Gravity :** Einstein claimed that the general relativity should be related to the gravitational potential. The reason why he insisted its relation with the gravity is because he made a strange assumption that g^{00} should be related to the gravitational potential ϕ as

$$g^{00} \simeq 1 + 2\phi \tag{4.4}$$

even though the g^{00} represents a coordinate system while the ϕ is a dynamical variable. Further, there is no way to confirm the above equation because the metric tensors are unknown functions which should be determined from the Einstein equation as a solution.

4.2 Planet Motion

Newton equation can describe quite successfully the motion of planets. For example, the earth revolves around the sun, and the earth orbit can be determined from the Newton equation perfectly. In this case, there is a gravitational potential whose strength is proportional to the mass of planets. Since the mass of the sun is much larger than any other planets, the center of gravity of the solar system is located on the sun.

4.2.1 Motion on Plane

The orbit of the earth revolution is found on the plane, and the reason why it is on the plane can be clearly answered from the Newton equation. This is because the gravitational potential is a central force in which the angular momentum $\boldsymbol{L}$ of a particle is conserved. That is,

$$\frac{d\boldsymbol{L}}{dt} = 0. \tag{4.5}$$

In this case, the motion $\boldsymbol{r}$ of a particle is orthogonal to the angular momentum

$$\boldsymbol{r} \cdot \boldsymbol{L} = \boldsymbol{r} \cdot (\boldsymbol{r} \times \boldsymbol{p}) = (\boldsymbol{r} \times \boldsymbol{r}) \cdot \boldsymbol{p} = 0. \tag{4.6}$$

Therefore, if we fix the direction of $\boldsymbol{L}$ to the $z-$axis, then we find that the motion $\boldsymbol{r}$ should be on the $x - y$ plane.

4.2.2 Ellipse Orbit

The orbit of the earth revolution is an ellipse. In reality, the earth has some interactions with other planets like Jupitar, and therefore, the earth motion must be slightly more complicated than the simple ellipse. However, as far as one period of revolution is concerned, the earth orbit should be indeed considered to be an ellipse. Now, the Newton equation with the gravitational potential is written as

$$m\ddot{\boldsymbol{r}} = -\frac{G_0 M m}{r^3}\boldsymbol{r} \tag{4.7}$$

which can be easily solved, and the solution of the orbit becomes

$$r = \frac{A}{1 + \varepsilon \cos\varphi} \tag{4.8}$$

where

$$A = \frac{\ell^2}{G_0 M m^2}, \quad \varepsilon = \sqrt{1 + \frac{2E\ell^2}{m(G_0 M m)^2}}. \tag{4.9}$$

Here, ℓ and ε denote the angular momentum and eccentricity, respectively, and A is related to the orbit radius. The observed value of the eccentricity of the earth revolution is $\varepsilon = 0.0167$, and thus, the earth orbit is almost a circle.

4.2.3 Observable is Period

In Kepler motion, its observable is a period of the orbit. In the earth revolution, its period is about 365.24 day. In classical mechanics, the earth orbit itself becomes an observable, and in fact, its average radius and eccentricity are observed. The relativistic effect on the Kepler motion is seen in the period and can be observed. However, this effect on the average radius of the orbit must be practically impossible to observe. This point should be discussed in the later section.

4.2.4 Interactions between Jupitar and Other Planets

The eccentricity of Mercury is $\varepsilon = 0.21$, and therefore, it is a real ellipse. In this case, if one draws a geometrical figure of the orbit of Mercury, one sees clearly the perihelion of the ellipse. Therefore, people in the 19 century were interested in the Mercury perihelion shift due to the effects of the other planets. The first elaborate calculations were done by Le Verrier in 1859, and after that, Newcomb carried out the same calculations and people refer to the result of his calculation.

The observed value of the Mercury perihelion shift is cited as 5599.76 arcsecond by Newcomb. However, the reliability of this value has not been examined properly. From this value, Newcomb subtracted his calculated result, and he claimed that the Mercury perihelion shift of 43 arcsecond is left unexplained. This value of the order 10^{-7} is very small, and it should be quite important to reexamine the observed value of the Mercury perihelion shift.

4.2.5 Extrasolar Asteroid (1I/2017 U1)

Recently, there is an interesting observation of extrasolar asteroid which is known as "1I/2017 U1 (Oumuamua)". Its diameter may be around 200 m.

The reason why this asteroid is not trapped by the sun should be because its velocity is quite high. The velocity v_o at long distance is considered to be

$$v_o \simeq 0.9 \times 10^{-4}\, c \tag{4.10}$$

which is almost the same as the earth revolution velocity of $v_e \simeq 1.0 \times 10^{-4}\, c$. Further, its velocity near the sun should be around

$$v \simeq 2.9 \times 10^{-4}\, c \quad \text{(at perihelion point).} \tag{4.11}$$

This estimation can be done from the energy conservation

$$E = \frac{1}{2} m_o v_o^2 = \frac{1}{2} m_o v^2 - \frac{G_0 M m_o}{R} \tag{4.12}$$

where m_o and M denote the masses of the asteroid and the sun, respectively. R is the distance between the asteroid and the sun. When the asteroid came cross to the earth, its velocity was $v \simeq 1.7 \times 10^{-4}\, c$.

The evidence that this asteroid should have come from the outside of the solar system is due to its orbit which is hyperbola. Indeed, from the observation, it turns out that the eccentricity of this asteroid is

$$\varepsilon_o \simeq 1.19. \tag{4.13}$$

Here, we should write the hyperbola in polar coordinate

$$r = \frac{A}{1 + \varepsilon_o \cos\varphi} \tag{4.14}$$

which is, of course, the same as the Kepler solution. Here, there is a constraint that the denominator of eq.(4.14) should be always positive and thus

$$|\cos\varphi| < \frac{1}{\varepsilon_o}. \tag{4.15}$$

Therefore, the asteroid must have come from the outside of the solar system.

4.2.6 Collision Between Milky Way and Andromeda

We live in the Milky Way, and the galaxy next to us is Andromeda. The Andromeda and Milky Way should eventually collide with each other. The time when they collide can be easily estimated, and here, we should only give the result. The two galaxies are approaching to each other with the velocity of about 1.2×10^5 m/s. Therefore, we can calculate the time when they collide, and the result is after 3×10^9 year from now.

4.3 New Gravity Theory

Here we should briefly describe the new gravity model. This is a field theory model, and therefore, readers may well have some difficulties to understand the new gravity model. In field theory, we always start from the Lagrangian density. Therefore, we should first write the Lagrangian density $\mathcal{L}$ of gravity model as well as electromagnetic interactions. Also the Lagrangian density $\mathcal{L}_w$ of the weak interactions with weak vector bosons is written here

$$\begin{aligned} \mathcal{L} &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - e\bar{\psi}\gamma^\mu\psi - m(1+g\mathcal{G})\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\mathcal{G}\partial^\mu\mathcal{G} \\ \mathcal{L}_w &= \bar{\Psi}_\ell(i\partial_\mu\gamma^\mu - m_q)\Psi_\ell - gJ_\mu^a W^{\mu,a} + \frac{1}{2}M^2 W_\mu^a W^{\mu,a} - \frac{1}{4}G_{\mu\nu}^a G^{\mu\nu,a} \end{aligned}$$

where $F_{\mu\nu}$ denotes the field strength of electromagnetic interaction

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

$G_{\mu\nu}^a$ is the field strength of the weak field and is defined as

$$G_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a$$

where W_μ^a denotes the field of weak bosons.

• **Lagrangian Density of Quantum Chromodynamics :** In this short lecture, we do not write the Lagrangian density of QCD (Quantum Chromodynamics) because it can be found in any field theory textbooks. For this QCD, we should make a brief comment. QCD is a non-abelian gauge theory which has a special property. There is a strong constraint on the dynamics of quarks and gluons from the kinematics of SU(3) group theory. One can easily prove that the color charges of quarks and gluons are gauge dependent, and thus, these particles are not observables. Therefore, quarks and gluons cannot be free, and thus they are confined. In this case, since there are no free quarks and gluons, we cannot make a perturbation scheme, and thus, there is no way to calculate any physically meaningful quantities in QCD.

4.3.1 Poisson Equation of Gravity

From the Lagrangian density, we can obtain the equation of motions. But this procedure is not so easy, and thus, we do not discuss it here. Instead, we should write only the results. From the Lagrange equation, we obtain

$$\boldsymbol{\nabla}^2 \mathcal{G}_0 = Mg\rho_g = Mg\delta(\boldsymbol{r}) \tag{4.16}$$

which is the gravitational Poisson equation. From this equation, we see that the gravity is indeed attractive. Here, we assume that proton is at rest with its density of $\rho_g(\boldsymbol{r}) = \delta(\boldsymbol{r})$. In this case, the solution of eq.(4.16) can be easily obtained as

$$\mathcal{G}_0(\boldsymbol{r}) = -\frac{Mg}{4\pi r}. \tag{4.17}$$

Therefore, the interaction Hamiltonian H_g of gravity becomes

$$H_g = mg\mathcal{G}_0(\boldsymbol{r}) = -\frac{G_0 mM}{r} \tag{4.18}$$

where $G_0 \equiv \frac{g^2}{4\pi}$ denotes the gravitational constant which should be determined from experiment. This potential proves that the inertial mass and gravitational mass are just the same. In comparison with the electromagnetic interaction, g plays the same role as e which couples to each interacting particle. But in the case of gravity, the mass m also couples to each interacting particle, and this is the basic and important difference from the electromagnetic interaction.

4.3.2 Period of Earth Revolution

In the Kepler problem, the most reliable physical observable must be a period of revolution. In fact, the period of the earth revolution is measured very accurately, and this is predicted quite well by the Newton equation. The period of the earth revolution is about 365.24 day.

The correction to the period of the earth revolution due to the general relativity is calculated, and it is found to be

$$\Delta T \simeq -3.6 \ \text{ second/year} \tag{4.19}$$

which predicts that the period must become shorter as a correction. This advance shift of the period correction comes from the attractive interaction arising from the general relativity correction. Here, a serious question may arise as to why experts of the general relativity should not have proposed to measure this effect of 3.6 second/year since this must be well measurable.

• **Leap Second and New Gravity Model Prediction :** The direct measurements of the period correction of the earth revolution have not been reported yet. But the indirect measurement of the period correction is known in terms of the leap second correction. People started to make a correction of one second per year in 1972, and they have made the correction of 25 seconds for 40 years until now. Thus, this measured correction amounts to

$$\Delta T \simeq 0.625 \ \pm 0.013 \text{ second/year} \tag{4.20}$$

which is a delay of the period. On the other hand, the new gravity model predicts the period correction as

$$\Delta T \simeq 0.63 \ \text{ second/year} \tag{4.21}$$

which is a delay. This prediction agrees very well with the observation.

From this comparison of the period corrections between the general relativity and observation, we can conclude that the general relativity is completely excluded as a theory of gravity. In reality, however, the general relativity has never been applied to any field of researches in modern physics, and therefore, there should remain no influence on theoretical physics at all.

4.3.3 Gravitational Wave

It is really a shame as a theoretical physicist that we have to make a brief explanation about the gravitational wave. It is beyond imagination that some group of people insisted that they observed a signal of the gravitational wave. Those people who claimed a "discovery" of the gravitational wave should be far from physicists, and their standard of understanding physics must be lower than the fourth grade student of university.

When a physical object can propagate in vacuum, then it must be a particle like photon whatever it may be, even though massless. This is confirmed from the vast amount of experiments, and by now, "the ether hypothesis" is completely excluded. In fact, all modern physics is based on the relativity principle, and there is no experiment which contradicts the relativity.

Therefore, unless the "graviton" were measured, any gravitational wave cannot be observed. However, the strength of gravity is 10^{-35} times smaller than the strength of electromagnetic force, and therefore, it is simply impossible to measure the graviton. This is well known to physicists and therefore, generally, any physics text books do not make the explanation of the gravitational wave.

4.4 Cosmology

Constituents of universe are basically proton, electron and photon. Neutrino should exist in universe, but at present, we do not know its role in cosmology. Neutron should be stable in nucleus and thus it can be a constituent. However, if it decays, then it becomes proton, electron and anti-neutrino, and therefore, we should not refer to it any more.

The most important point is that these particles are stable. In cosmology, it means that the universe should have existed since the infinite past time, and this is the basic principle when developing the theory of cosmology. It is clear that the big bang model cannot be a subject of scientific discussion since it neglects this fact of particle stability.

4.4.1 Constituents of Universe

Proton and electron are Fermi particles, and therefore, this number should be preserved. This means that this universe must have existed from the infinite past time, and therefore it should exist in eternal future. This is the starting point in the discussion of cosmology.

4.4.2 Star

Proton and electron should attract each other and make a hydrogen atom. Two hydrogen atoms get together and become a hydrogen molecule. This hydrogen molecule must be the basic substances in the universe in the course of forming stars.

In the universe, there should be a group of hydrogen molecules which is called a molecular cloud. The bulk materials of hydrogen molecule should be the basic ingredients of star formation. If the mass of prototype materials should be of the same order to the sun, then the nuclear fusion should eventually start. This fusion is a reaction of four protons into He nucleus together with two positron and two neutrinos, as a result.

If the mass of stars should be several times larger than that of the sun, then the temperature of the center of star should become sufficiently larger, and thus three He nucleus start to fuse into ^{12}C nucleus. Once ^{12}C is created, then He nucleus should collide with ^{12}C successively to make heavier and heavier nuclei like ^{16}O, ^{20}Ne and so on. In fact, the observed nuclear abundance indicates that these nuclei of He multiples exist more than other nuclei in the universe.

4.4.3 Galaxy

The formation of galaxy cannot be understood very clearly at present. In order to understand the galaxy, then we may have to go back to the formation of the universe itself. In this galaxy, it is believed that there should be $\sim 10^{11}$ stars. Therefore, the galaxy nucleus must be some kind of quasi-star that should have a gigantic amount of mass.

4.4.4 Supergalaxy Universe

This universe should have a cluster of about $\sim 10^{11}$ galaxies and its size should be around $\sim 1.5\times 10^{10}$ light year distance. If the whole universe consists of only this cluster of galaxies, then this picture disagrees with infinite times of explosions and expansions. Therefore, we call this type of universe "supergalaxy universe" which has been repeatedly doing "explosions and expansions". In this way, it should be a natural assumption that there must exist an infinite number of supergalaxy universes.

In this case, a question may arise as to how we may observe some evidence of supergalaxy universe. At present, we do not have any physically plausible way to measure its distance, and there is practically no possibility to observe any signal from the supergalaxy universe.

There might be a possibility to observe some evidence of its existence in case there is a supergalaxy universe nearby which is just in the early stage of explosions. In this case, this supergalaxy universe should be emitting the vast amount of light. In addition, these objects that emit photons should be moving relativistically, and thus the light we observe must have a strong "blue shifts". If we could observe any kind of blue shift light, then there might be some chance to observe some signal from supergalaxy universe.

4.4.5 Mugen Universe

Since there must exist an infinite number of supergalaxy universes, we should call this whole universe "Mugen universe". The size of the Mugen universe should be infinite, and it should exist from the infinite past time to the eternal future.

- **Question : Why infinity?**

Evidence 1

What should be the ground that Mugen universe is infinite? If the number of supergalaxy universes is finite, then due to the gravitational force, they should eventually fuse into one piece of matter. But this contradicts the fact that universe should have existed from the infinite past because of the stability of proton and electron. In fact, we can prove that, if the universe is infinite, then each supergalaxy universes should be stable.

Evidence 2

The infinite size and infinite number of supergalaxy universes in Mugen universe should be related to the background radiation. Suppose that the supergalaxy universe should make the explosion and expansion. Then, it should lose a large amount of energy in terms of photon emission at one explosion. If this has been repeatedly happening in the supergalaxy universe, then it should have lost whole energy completely. This contradicts the present stage of supergalaxy universe, and thus it should obtain some energy from outside. This should correspond to the background radiations. Therefore, it is quite natural that the Mugen universe inevitably supports the existence of the background radiation.

Further, the explosions and expansions of the supergalaxy universe can occur only when it is in the "background radiation cradle". In this respect, the background radiation should be a crucial element of the Mugen universe.

4.5 Homework Problems

Answer the following questions with your own words.

4.1 The falling pattern of a body on the moon should not depend on its mass. Why is it so? Answer for this question intuitively.

4.2 Compare the strength of gravitational force on the earth and moon. Here, the force F is given as $F = -\frac{GMm}{r^2}$. The masses and radii of the earth and moon are given, respectively, as
$M_e = 5.97 \times 10^{24}$ kg, $R_e = 6378$ km
$M_m = 7.34 \times 10^{22}$ kg, $R_m = 1737$ km

4.3 The Coulomb and gravitational potentials should have the shape proportional to $1/r$. Why is it so?

4.4 In the electromagnetic fields, there is photon, but in the gravity, there is no graviton. Even if it were there, there is no way to observe it. Why is it so?

4.5 The general relativity is a theoretical model for the coordinate system. What do you think is the physical meaning of equations for the coordinate system? Answer for it with your own words.

4.6 Explain the physical meaning of inertial and gravitational masses.

4.7 In electromagnetisms, particles should have a charge as a quantum number. What about the gravity case? Do particles have a corresponding physical quantum number similar to a charge?

4.8 All the planets in the solar system should revolve on the same plane. Make up your own picture how the solar system was created.

4.9 In the electromagnetic interaction, the coupling constant is e. What about the gravity case?

4.10 A free neutron should decay into proton, electron and anti-neutrino. Why are neutrons in nucleus stable?

4.11 The element abundance of the sun is mostly hydrogen. On the other hand, the abundance of the planets should consist of many other elements than the hydrogen. Why is it so?

4.12 In order that the living organism should exist on the planet, there must be water and air. In this case, the planet should have its mass which is larger than a certain value of mass. Why is it so?

4.13 Atmosphere is basically the weight of air, and this is due to the gravitational attraction to air. In contrast to liquid, air can survive not only on the surface of the earth, but also in the sky. Why is it so?

4.14 Liquid should exist when the gravitational force is above a certain level. What should happen when liquid is in outer space?

4.15 Estimate roughly the velocity of the earth revolution. Here, the radius R of the earth revolution is $R \simeq 500c$, and light velocity is $c = 3 \times 10^8$ m/s. Further the period T of revolution is $T \simeq \pi \times 10^7$ s.

4.16 If we assume that this universe is infinite, then it may contradict the Olbers paradox. Examine whether Mugen universe might contradict the Olbers paradox or not.

4.17 Newton equation on the surface of the earth should be affected from the earth's rotation. One of the effect is the centrifugal force. What kind of effect should you expect from this centrifugal force?

4.18 Newton equation on the surface of the earth should be affected from the earth's rotation. One of the effect is the Coriolis force. Explain why typhoon vortex is always made in the counterclockwise way.

• **Typhoon Vortex :** Due to the Coriolis force, Typhoon vortex is always made in the counterclockwise way.

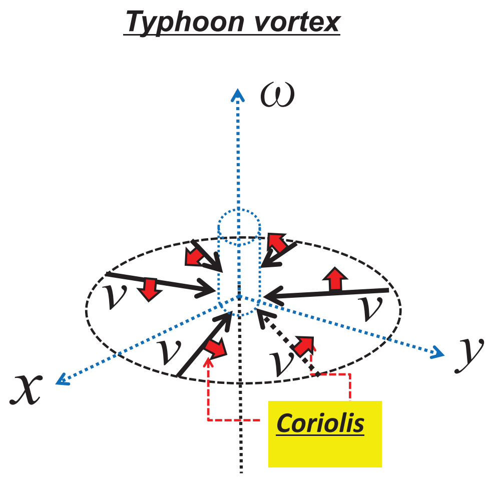

Figure 4.4: Typhoon Vortex

Chapter 5. Black Hole for Dilettante

Even now, some articles about Black Hole are often found in mass media. Unfortunately, however, most of those authors of the articles do not understand the basic physics of Black Hole at all. This must be mainly because they are not theoretical physicists, and further those people that provide news sources of the articles must be, indeed, far from experts not only on quantum field theory but also on general relativity.

In this short note, I should make a brief but scientifically correct explanation concerning the physics of Black Hole. At present, there are so many fake information on the physics of Black Hole that some reliable physics lectures must be absolutely needed to improve the present situation. For example, some group of scientists insisted that they discovered Black Hole by making photograph of nucleus of galaxy. This is practically a joke in physics and is quite similar to the story that a man insisted to have seen a god in the forest.

Why can these incredible stories be floating around in mass media? This must be because these writers of scientists do not understand the modern physics at all. Nevertheless, if these stories are repeatedly reported in the mass media, then non-physicists might well think and accept that the existence of Black Hole might have been confirmed. This is unfortunate since they do not know what Black Hole should be in physics terminology.

5.1 What is Black Hole ?

Recently, quite a few people have asked me to explain what should be the physics of Black Hole. Therefore, I decided to make a brief but reliable explanation as to what should be the physical meaning of Black Hole in terms of modern physics terminology.

At present, Black Hole is considered to be a kind of star, but its original definition comes from the singularity of the special solution of the Einstein equation. In this sense it has nothing to do with stars. However, those people who consider themselves to be experts on Black Hole may have a picture similar to neutron stars even with higher density. But they believe that light could not escape from the boundary of Black Hole.

• Black Hole in Space-Time :
Now, it is believed that Black Hole is a hole in space-time which is "black", even though they do not understand what it means by "black" in terms of physics. In addition, "experts" on Black Hole do not understand the dynamics of Black Hole at all since they are just physicists who cannot carry out any physical evaluation of neutron stars. In fact, they just talk about Black Hole with their imagination, which has nothing to do with science. Therefore, concerning the story of Black Hole, most of people dilettante are just bound in the chaotic states for a long time.

5.1.1 Nucleus of M87 Galaxy

The recent observation of a would-be Black Hole is related to the nucleus of M87 galaxy. This galaxy is in the distance of 60 million light years from here and it has presumably a diameter of 120 thousand light years. Further it may have a nucleus of galaxy which has a 650 million solar mass. Apart from the accuracy of these numbers, it should be quite natural that the nucleus of galaxy should have some kind of neutron stars, and this is not inconsistent with modern physics. In this case, one can easily estimate the radius of this neutron star, and it is around 10 thousand km which is slightly larger than the earth radius.

5.1.2 Black Hole and Neutron Star

From the kinematics of Black Hole, light cannot escape from the surface of this Black Hole, and this is the basic assumption of Black Hole, though without any physical foundation. This is the only point that is connected to the difference between neutron star and Black Hole. Thus, one can easily see that there is no way to observe this difference between neutron star and Black Hole.

5.1.3 Formation of Super Neutron Star

The formation of super neutron stars should be connected to the large supernova explosion. This type of formation mechanism must be very important to understand, but it has never been studied until now. This may well be connected to the fact that the new gravity model is discovered only about ten years ago [4, 5], and therefore, it is clear that the dynamics of nucleus of galaxy should now be investigated. Indeed, it must be a very interesting subject in nuclear astrophysics.

5.1.4 Responsibility of Person in Charge in Science Section ?

It is a serious problem that mass media published many exaggerated and fake articles which reported that Black Hole was discovered. However, the responsibility for writing incorrect stories of Black Hole may not necessarily be held by people in charge in science section of mass media. But it may well be that the real responsibility of writing wrong articles should rather be taken by the physicists that distributed publicly these incorrect information on Black Hole. Unfortunately, these physicists understand neither modern physics nor general relativity, and probably the fact that they are “physics amusia” must be much more serious than the propagation of wrong information about Black Hole.

5.1.5 Black Hole and Neutron Star

In order to clarify the physics of Black Hole one must understand quantum field theory, astrophysics, nuclear physics and general relativity in depth, and further one should be able to calculate some physical quantities in this field of research.

• **Gravitational Collapse ? :**
For example, nuclear physicists should know quite well that the nucleon-nucleon interactions should be strongly repulsive at the short distance, and thus they know that any stars with much higher density than neutron stars cannot exist at all. On the other hand, any large stars with similar density as neutron stars may well exist in nature since there is no basic problem for the formation of gigantic neutron stars. In addition, there is no physical process of gravitational collapse since the gravitational force cannot be very large at the origin of neutron star center. In fact, the gravitational force with finite distribution of mass has the strength which is only 1.5 times stronger at the origin than at the surface.

5.2 Physics of Black Hole

Historically, Black Hole is defined as the singularity of the solution in the Einstein equation, and thus it has nothing to do with the formation of stars. Therefore, experts claim that Black Hole is a black "hole" in space-time or they assume that, near the surface of Black Hole, space is distorted so that light cannot escape from Black Hole or something of this kind. These explanations have nothing to do with physics, and therefore only the terminology of Black Hole have been floating around until now. Unfortunately, Black Hole became very popular as if it were a special kind of star.

5.2.1 Neutron Star

Experts may explain that Black Hole is a star which has a very high density, and they imagine that it should be similar to neutron stars, but should have even higher density than neutron stars. However, they do not discuss how Black Hole can be formed in the universe since there is no physics equation related to the general relativity. This is clear since the general relativity is a theory for the coordinate system. Therefore, it has nothing to do with dynamics, and indeed no dynamical model is related to the general relativity.

5.2.2 Nucleus of Galaxy

Black Hole has no relation with the internal structure of star, and experts define or only claim that Black Hole should have a very large density. However, stars with very high density are, of course, known as neutron stars which are confirmed in terms of Pulsars. In this respect, the nucleus of galaxy should be a very high density star similar to neutron star, and this is consistent with the modern physics. Therefore, it should not be surprising if the nucleus of galaxy becomes an enormous neutron star in size and mass. Indeed, it should attract billions of stars in galaxy, and therefore, it should not be strange at all if there should be a gigantic neutron star at the center of galaxy.

5.2.3 Surface of Black Hole

The most important assumption concerning Black Hole is related to space distortion at the surface of Black Hole, and it is assumed that light cannot escape from Black Hole. However, space distortion in three dimensions cannot be physically understandable at all. Space distortion is replaced by the light propagation in space, but this cannot be treated in terms of classical mechanics. Further, the general relativity is not a dynamical theory, and therefore, it cannot make any predictions how light should propagate in space. "Experts" on Black Hole only state verbally but not physically as to how space should be distorted, though only from their imagination. The propagation of photon can be treated only if the electromagnetic field is quantized. In addition, space in the general relativity is just the coordinate system, and human being cannot realize real space.

5.2.4 Space Distortion is a Prank of "Physics Amusia"

Thus, nobody can understand space distortion at all, and those people who draw some picture of space distortion are simply making their imagination of scientific fiction. The idea of space distortion must be a result of a prank from "physics amusia", and it has nothing to do with physics.

5.3 Einstein and General Relativity

Here, there is no important reason to make any tutorial description of the general relativity since the model is worthless in physics. The general relativity is an equation for coordinate system, and Einstein thought that the coordinate system might well be influenced if there should exist a distribution of stars. This is obviously a model which is constructed by physics dilettante. Further, this general relativity is not consistent with the special relativity even though the relativity principle is the most important physics law. Probably, Einstein might have realized this fact of violation of the special relativity, and therefore, he may have claimed that the relativity should be called "special relativity" and his new theory should be named "general relativity".

5.3.1 Relativity and Its Importance

Most readers may well tend to think that the work of theory of relativity must have been achieved mainly by Einstein. However, it is, by now, known to experts that the credit of constructing the theory of relativity may not necessarily go to Einstein, and his contribution to the relativity should be carefully re-examined.

• **Rest Mass :**

Indeed, the connection of the rest mass with the Lorentz invariant quantity is an important achievement made by Einstein. However, the real importance of theory of relativity should not be for this rest mass issue, but for the theoretical framework itself that all the theoretical models must satisfy the Lorentz invariance. This is, of course, quite well-known to modern physicists.

In fact, this formulation of relativity is made up by Lorentz and Minkowski and other scientists before Einstein, and therefore, Einstein's contribution to the theory of relativity is not necessarily very great.

• **Overvalued :**
At present, the work of Einstein concerning theory of relativity is considered to be overvalued. In fact, his paper has no reference, and thus it is written as if everything were done by himself. This is not a fair way of writing papers, but at the time of his day, this way of writing might be one of the reasons of overvaluation.

• **General Relativity is Inconsistent with Relativity :**
On the other hand, since the general relativity does not satisfy the transformation property of relativity, it is quite difficult to accept that Einstein understands the essence of theory of relativity. In this sense, readers may well understand that it is simply impossible to appreciate the general relativity from modern physics point of view.

5.3.2 Fundamental Equation in Physics

If one wishes to construct a fundamental equation in physics, then one has to make all kinds of careful examinations of physical phenomena from various aspects. However, the Einstein equation is just constructed by making the second order differentials of the metric tensor in the left-hand side and by making the energy- momentum tensor with the distribution function of stars in the right-hand side.

• **Physical Ground of Einstein Equation :**
Surprisingly and frighteningly, however, there exists no physical phenomenon corresponding to its basic ground of the Einstein equation. Furthermore, one cannot understand what the equation for the coordinate system means in physics. Probably, the author of this equation by himself should not have any concrete pictures for the equation, apart from the vague imagination of space distortion. At the end of 19 century, there seems to be a paper which discusses space distortion, and possibly Einstein may have referred to this paper.

5.3.3 Physics Sense of Einstein

Up to now, the general relativity is critically reviewed, but no special comment on Einstein himself is made yet. Here, however, I should make a brief comment on Einstein himself even though this is nothing but a feeling. It is not clear whether Einstein might be a "physics amusia" or not, since, at the time of construction of general relativity, quantum mechanics was not discovered yet. Therefore, it is not surprising that he did not have any quantum mechanical and probabilistic way of thinking at the time of 1917.

• **Solvay Conferences and Controversy of Quantum Mechanics :**
However, the controversy of quantum mechanics between Bohr and Einstein at the Solvay Conferences on Physics in 1930 indicates that Einstein could not understand the essence of quantum mechanics which is a probabilistic behavior. This may suggest that Einstein continued to keep the deterministic view of the world, and he wanted to defend the general relativity that is the center of this ideology. By now, it is confirmed that the fundamental physics is described in terms of quantum and probabilistic pictures. On the other hand, some group of physicists still believe in the general relativity, even though it disagrees with quantum behavior. What should be their aim?

5.4 Physics and Shokunin (Professionals)

If one wishes to achieve something interesting in physics, then one should become "physics Shokunin". In order to become a theoretical physics Shokunin, for example, one should solve all kinds of exercise problems and examine fundamental physics formulation, in particular, Dirac equation with electromagnetic interactions.

5.4.1 Importance of Shokunin

The Shokunin spirit must be important for other area of researches as well. Japanese should have a respect for Shokunin spirit since Edo period. This spirit may well be similar to the Meisterschaft in Germany. In fact, it is believed that this spirit must have been a key issue for the cultural and economic developments of Japan and Germany.

5.4.2 Drastic Decrease of Theoretical Physics Shokunin

In the field of theoretical physics, however, Shokunin researchers have decreased drastically. This may be related to the fact that many researchers at present tend to become knowledge-biased, and therefore, they do not work hard to improve their skills in physics. However, even if they transpose a knowledge of one field of research into the other field, this does not mean that they make any real progress in physics. In order to make a solid progress even a little bit, one has to work very hard to improve one's skills of theoretical and computational as much as possible.

• **All Physics Institute :**
At present, a few researchers work at "All Physics Institute" to make a real progress in physics, though they are materially impoverished but physically quite rich indeed. This group of Shokunin researchers are reconstructing modern physics, and a new theoretical scheme will be eventually constructed in near future.

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