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A Chain of Mistakes in Theoretical Physics

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Preface

For a long time, a chain of mistakes have been made in theoretical physics. Even though it is neither interesting nor encouraging to explain the stories of wrong model calculations, it should be important to clarify underlying reasons as to why people made mistakes in their calculations. In addition, we should make it clear why people accepted such incorrect model calculations over the years.

In this short note, I should like to briefly describe a chain of mistakes in theoretical physics. First, I begin with discussing the anomaly problem which is proposed by Adler, and then clarify there should be no physical meaning in the Einstein equation. Further, the problem of Feynman propagator is explained why it should not be applied to the loop diagrams, and the problem of Dimensional Regularization by 't Hooft and Veltmann is also clarified. In addition, the intrinsic defect of Weinberg-Salam model is clarified in connection with the non-abelian nature of gauge fields.

Finally, I should make a brief comment on the confinement of quarks and gluons since people misunderstood the confinement of quarks as the dynamical effects. In reality, the confinement of quarks should be due to the kinematical effects in that the color charges of quarks are not physical observables.

Most of the papers discussed in this short note should be found in the references of "Fundamental Problems in Quantum Field Theory" (Bentham Publishers, 2013).

In Appendix, I should discuss some old topics which should be reexamined from the point of view of new theoretical scheme. These descriptions of the topics may help young physicists understand modern physics in depth.

Contents

Chapter 1

Adler's Anomaly

In 1969, Adler presented the T-matrix calculation of triangle diagrams in order to explain the decay processes of $\pi^0 \to 2\gamma$ and/or $Z^0 \to 2\gamma$. In his calculation, however, he made a serious mistake in the evaluation of T-matrix of $Z^0 \to 2\gamma$. For this reason, he believed that this T-matrix of $Z^0 \to 2\gamma$ should contain a linear divergence, and thus he invented a new regularization method of linear divergence. From this new scheme, he discovered the anomaly equation by regularizing this linear divergence.

On the other hand, just before Adler's calculation, Nishijima made a careful calculation of the triangle diagrams that correspond to the decay process of $\pi^0 \to 2\gamma$. This calculation of triangle diagrams must be one of the most important and impressive evaluations of modern field theory applications. Indeed, the calculated result of $\pi^0 \to 2\gamma$ process can reproduce the observed life time of π^0 remarkably well.

1.1 Triangle Diagrams

In this section, we should explain why and where Adler may have made mistakes in evaluating the triangle diagrams [2]. First, we consider the T-matrix evaluation of triangle diagram of $Z^0 \to 2\gamma$ decay process. In this case, the momenta of two photons can be denoted as k_1 and k_2, and the scattering T-matrix can be written as [3]

$$\begin{aligned} T_{AVC} \simeq\ & e^2 g_z \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[(\gamma\epsilon_1) \frac{1}{\not{p} - M + i\varepsilon} (\gamma\epsilon_2) \right. \\ & \left. \times \frac{1}{\not{p} - \not{k}_2 - M + i\varepsilon} \gamma_5 (\gamma\epsilon_v) \frac{1}{\not{p} + \not{k}_1 - M + i\varepsilon} \right] + (1 \leftrightarrow 2) \qquad (1.1) \end{aligned}$$

where M denotes the mass of fermion. ϵ_1 and ϵ_2 are the polarization vectors of two photons while ϵ_v^μ corresponds to the polarization vector of Z^0 boson.

From eq.(1.1), it seems that the T-matrix may induce the linear divergence at a glance. However, one can rigorously prove that the sum of $(1 \leftrightarrow 2)$ terms on the T-matrix should vanish to zero before the integration over d^4p, and therefore, there is no infinity present [4].

1.1.1 Disappearance of Linear Divergences

It should not be very easy to sum up the two Feynman diagrams so as to prove that the sum should vanish to zero. However, it is rather easy to show that there should not be any linear divergence terms in the triangle diagrams. In fact, we should look at the leading term of p by expanding in terms of $1/p$. In this case, the T-matrix becomes

$$\mathrm{Tr}[\not{p}\gamma_\mu\not{p}\gamma_\nu\not{p}\gamma_\rho\gamma_5]\epsilon_1^\mu\epsilon_2^\nu\epsilon_v^\rho + \mathrm{Tr}[\not{p}\gamma_\mu\not{p}\gamma_\nu\not{p}\gamma_\rho\gamma_5]\epsilon_2^\mu\epsilon_1^\nu\epsilon_v^\rho = 0 \tag{1.2}$$

where we make use of the following identity equation

$$\mathrm{Tr}[\not{p}\gamma_\mu\not{p}\gamma_\nu\not{p}\gamma_\rho\gamma_5] = -\mathrm{Tr}[\not{p}\gamma_\nu\not{p}\gamma_\mu\not{p}\gamma_\rho\gamma_5]. \tag{1.3}$$

Therefore, it is proved that the T-matrix [eq.(1.1)] does not have any linear divergence terms, and thus it has nothing to do with the integration over momentum.

1.1.2 Landau-Yang Theorem

As we see above, the sum of the two Feynman diagrams should vanish to zero. Therefore, the T-matrix of eq.(1.1) does not have any linear divergences, and this fact should be related to the Landau-Yang theorem [5, 6]. This theorem states that any massive particle with spin of 1 cannot decay into two photons due to the spin selection rule. This theorem can be proved in the following way. The total angular momentum states which can be made from two 1^- states can be written as

$$\begin{aligned}\boldsymbol{Y}^1(\Omega_1)\otimes\boldsymbol{Y}^1(\Omega_2) &= (\boldsymbol{Y}^1(\Omega_1)\cdot\boldsymbol{Y}^1(\Omega_2)) \oplus [\boldsymbol{Y}^1(\Omega_1)\otimes\boldsymbol{Y}^1(\Omega_2)]^{(1)} \oplus \\ &\quad [\boldsymbol{Y}^1(\Omega_1)\otimes\boldsymbol{Y}^1(\Omega_2)]^{(2)}\end{aligned} \tag{1.4}$$

which is the consequence of the group theory calculation. In the right hand side, the first term 0^+ and the third term 2^+ should be excluded from the present discussion. The main interest must be the second term which has a spin of 1. However, this term in eq.(1.4) is the anti-symmetric state in terms of exchange of 1 and 2. On the other hand, the two photon state must be symmetric with the exchange of 1 and 2. Therefore, this second state in eq.(1.4) cannot be made from two photons, and this corresponds to the Landau-Yang theorem. To summarize, any massive particles with its spin of 1 cannot decay into two photons due to the spin selection rule which is expressed as the Landau-Yang theorem.

1.2 Mathematical Misunderstanding

Here, we should clarify why Adler overlooked the cancellation between two terms in eq.(1.1). This is simply a mathematical mistake, and we should explain the reason why he made a wrong conclusion in the evaluation of the triangle diagrams. This is quite important since this mistake should have led him to the discovery of the anomaly equation.

In order to understand the linear divergence term, we write the scattering matrix of T_{AVC} in a simplified expression as

$$T_{AVC} \simeq \int \frac{d^4p}{(2\pi)^4} \left[\frac{p_\mu (a_1^\mu - a_2^\mu)}{(p^2 + (b_1 - b_2)^2)^2} + (1 \leftrightarrow 2) \right]. \tag{1.5}$$

It is clear that this T_{AVC} should vanish to zero before the integration over momentum.

1.2.1 Adler's Mistakes

At this point, Adler made use of the integration formula in a wrong way, and we should explain it more in detail. We can write down the linear divergence integration in terms of one dimensional integration as

$$I = \int_{-\Lambda}^{\Lambda} dp \frac{p}{\sqrt{p^2 + c^2}}. \tag{1.6}$$

Under the transformation of $p \to -p$, this integral of I becomes

$$I = -\int_{-\Lambda}^{\Lambda} dp \frac{p}{\sqrt{p^2 + c^2}} \tag{1.7}$$

where the negative sign appears simply because the integral I must be zero. This means that eq.(1.7) should hold only because the integral itself should be zero. Now, if we consider the following integral, for example

$$J = \int_0^{\Lambda} dp \frac{p}{\sqrt{p^2 + c^2}} \tag{1.8}$$

then, this integral becomes under the transformation of $p \to -p$

$$J = \int_{-\Lambda}^{0} dp \frac{p}{\sqrt{p^2 + c^2}} \tag{1.9}$$

which is a different integral from the original form of eq.(1.8). This means that the validity of eq.(1.7) comes from the fact that the integral itself is zero, as stated above.

1.3 Anomaly Equation

In his paper, Adler assumed that the second term of eq.(1.1) must be the same as the first term. Therefore, he regularized the linear divergence term in eq.(1.1) and obtained the anomaly equation. It is unclear whether the regularization method can be justified or not in terms of physics terminology. But it is, for sure, true that he obtained his anomaly equation from the vanishing term. It may well be that his anomaly equation was quite new to physicists, and therefore, it had been accepted by people, in particular, the referees of Journal of Physical Review at that time.

1.3.1 Conservation of Axial Current

Here, we should briefly explain the conservation of axial current. This conservation law should hold for the massless fermion system. For example, we consider the Lagrangian density of QED $\mathcal{L}(\psi, \partial_\mu \psi)$

$$\mathcal{L} = i\bar{\psi}\partial_\mu \gamma^\mu \psi - e\bar{\psi}\gamma_\mu \psi A^\mu - m\bar{\psi}\psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}. \tag{1.10}$$

Here, if we put $m = 0$ in the above Lagrangian density, then the Lagrangian density of eq.(1.10) should be invariant under the following chiral transformation

$$\psi' = e^{i\alpha\gamma_5}\psi. \tag{1.11}$$

It should be noted that if there is a mass term present, then the mass term behaves under the chiral transformation as

$$\bar{\psi}'\psi' = \psi^\dagger e^{-i\alpha\gamma_5}\gamma_0 e^{i\alpha\gamma_5}\psi \neq \bar{\psi}\psi \tag{1.12}$$

and thus the mass term is not chiral invariant as is well-known. Therefore, the massless fermion system for the QED Lagrangian density should have a conservation of the Noether current associated with the chiral symmetry

$$j_5^\mu = -i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \gamma_5 \psi = \bar{\psi} \gamma^\mu \gamma_5 \psi. \tag{1.13}$$

That is,

$$\partial_\mu j_5^\mu = 0. \tag{1.14}$$

This indicates that the conservation of axial vector current should be derived from the symmetry of the system, and thus this conservation law cannot be broken unless same external forces should be present. This type of conservation law cannot be violated by some mathematical methods, such as regularizations and so on.

1.4 Negative Legacy

It is clear that any physical law must not be violated by the regularizations which are simply mathematical tools. In this respect, the anomaly equation has left us a serious negative legacy.

In reality, the scattering T-matrix of eq.(1.1) is proved to vanish to zero before the momentum integration, and this is, of course, consistent with Landau-Yang theorem.

Chapter 2

Einstein's General Relativity

The Einstein equation is a differential equation for the metric tensor of $g^{\mu\nu}$. This metric tensor is defined when the Lorentz invariant quantity $(ds)^2$ is expressed in terms of generalized formula as $(ds)^2 = g_{\mu\nu}dx^\mu dx^\nu$. However, there is no special physical meaning in this generalization, and thus we cannot find any physics related to the metric tensor of $g^{\mu\nu}$. This problem of the general relativity has nothing to do with physics, but it is important in the science history. Therefore, we should explain why the general relativity was accepted to physicists for such a long time, even though it is a meaningless theory in physics.

2.1 Relativity Principle

Relativity principle should require that equations of motion in any inertial system should have the same form of differential equations, and, thus, all of the physical observables must be the same in every inertial system. This is the essence of the relativity, and nature can be understood in terms of four basic Lagrangian densities of electromagnetic, weak, strong and gravitational interactions. Indeed, all the field theory models satisfy the relativistic invariance of Lorentz transformation.

2.1.1 Lorentz Transformation

Let us consider the moving frame $S(t', x', y', z')$ which is moving with linear motion of constant velocity v along $x-$axis with respect to the rest frame $R(t, x, y, z)$. In this case, the requirement that the equation of motion must be equivalent to each other in both systems can be written in terms of

Lorentz transformation

$$x = \gamma(x' + vt'), \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right), \qquad y = y', \qquad z = z'. \tag{2.1}$$

2.1.2 Lorentz Invariance

This Lorentz transformation is the necessary and sufficient condition for relativity principle. However, if we consider only the invariance of Lorentz transformation, then there should be many other physical quantities. Here, we should discuss the small distance square of $(ds)^2$ in four dimensions, which is defined as

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2.$$

2.1.3 Minkowski Space

This $(ds)^2$ is introduced by Minkowski as a Lorentz invariant quantity

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2 \tag{2.2}$$

which is indeed invariant under the Lorentz transformation of

$$x = \gamma(x' + vt'), \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right), \qquad y = y', \qquad z = z'. \tag{2.3}$$

Minkowski extended mathematically $(ds)^2$ to

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2 \equiv g^{\mu\nu}dx_\mu dx_\nu \tag{2.4}$$

even though there is no physical reason for this generalization. In this case, dx^μ and dx_μ are introduced as

$$dx^\mu = (cdt, dx, dy, dz), \quad dx_\mu = (cdt, -dx, -dy, -dz). \tag{2.5}$$

Further, the metric tensor $g^{\mu\nu}$ is defined as

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

This extension of $(ds)^2$ is not incorrect. However, the naming of $g^{\mu\nu}$ as metric tensor is wrong since it is a dimensionless quantity and, therefore, it cannot be taken as any measure of space and time.

2.2 Risk of Generalization

It indeed makes sense that $(ds)^2$ can be taken as a test of Lorentz invariance, and it is also understandable that $(ds)^2$ is expressed in terms of eq.(2.4). However, it should be important to realize that this generalization is physically meaningless since $(ds)^2$ itself is far from any essential quantity in physics.

2.2.1 Invariance of $(ds)^2$

Here, we should explain some important point of $(ds)^2$. This $(ds)^2$ is certainly Lorentz invariant, but it is the result of the Lorentz transformation, and not the condition. In fact, there should be many other transformations that can make $(ds)^2$ invariant. This point is quite important since it is related to the essence of relativity. The theory of relativity is a theoretical frame work in which any equation of motion must be the same in any inertial system. The Lorentz transformation satisfies this necessary and sufficient conditions. On the other hand, $(ds)^2$ can serve as a sufficient condition of the relativity requirement, but it is not necessary.

2.2.2 Generalized Expression of $(ds)^2$

For a long time, people believed that the generalized expression of $(ds)^2$

$$(ds)^2 = g^{\mu\nu} dx_\mu dx_\nu \tag{2.6}$$

must be basic and essential for $(ds)^2$. This is, of course, an illusion. However, most of physicists may well have been trapped for a long time in a blind state, and this is quite unfortunate.

2.2.3 Physical Meaning of $g^{\mu\nu}$

In physics, the expression of (2.2) is essential, and it is impossible to find any physical meaning for the metric tensor of $g^{\mu\nu}$. Indeed, $g^{\mu\nu}$ must be mathematically all right, but it has no physical meaning, and it is just useless.

2.3 General Relativity

Einstein equation is the differential equation for this useless metric tensor $g^{\mu\nu}$, and therefore, we cannot find any physical meaning in this equation.

In fact, even if the metric tensor $g^{\mu\nu}$ becomes some function of space and time, there is no effect on the relativity. In case the $(ds)^2$ which is expressed by $g^{\mu\nu}$ in eq.(2.6) has lost the Lorentz invariance, we should make use of $(ds)^2$ as expressed in eq.(2.2). Therefore, there is no physical effect of $g^{\mu\nu}$ in nature at all.

This clearly shows that the Einstein equation has nothing to do with physics, and it is simply a mathematical equation which may help young people learn geometrical differential equation as an exercise problem.

2.4 Negative Legacy

It is a shame that we could not clarify 30 years ago, for example, that the Einstein equation has nothing to do with physics. Many young people wasted their time by learning this general relativity which is completely meaningless in physics. This is quite unfortunate and serious.

Incidentally, there was a claim at one point that the Mercury perihelion shifts could be described by the metric tensor which is, by hand, connected to gravity. However, this shift is identified by the discontinuity of Mercury orbit, and, therefore, this prediction is both physically and mathematically meaningless. In this sense, this claim may well be one of the worst theoretical predictions in physics.

Chapter 3

Feynman Propagator

For a long time, it is believed that the gauge field theory with renormalization scheme should be the basic and right theoretical model in modern physics. In fact, people even claimed that they could not accept any theoretical models if they were not gauge field theory. Indeed, in some period of time, the present author was just in the middle of chaotic states in which every theoretical model must be constructed by the gauge field theory.

However, Dirac claimed that there must be something wrong with the renormalization scheme if one finds some infinity in physical observables [8]. In this respect, we should clarify why his claim was ignored by most of the theoretical physicists even though his claim should be very reasonable.

In reality, the cause of the infinity in the vertex corrections should be due to the defect of the Feynman propagator. But, for whatever reason, this problem is completely forgotten away.

3.1 Electron Vertex Corrections

The vertex correction of electron $\Gamma^\rho(p,q)$ can be calculated by the third order perturbation theory in quantum electrodynamics, and the corresponding T-matrix can be written as

$$\Gamma^\rho(p,q) = -ie^3 \int \frac{d^4k}{(2\pi)^4} \left(\frac{g^{\mu\nu}}{k^2 - i\varepsilon} \right) \gamma_\mu \frac{1}{\not{q} - \not{k} - m_e} \gamma^\rho \frac{1}{\not{p} - \not{k} - m_e} \gamma_\nu \tag{3.1}$$

where all the physical quantities above can be found in reference [3], if necessary. Here, the Feynman propagator [10]

$$D_F^{\mu\nu}(k) = -\frac{g^{\mu\nu}}{k^2 - i\varepsilon} \tag{3.2}$$

is employed. From eq.(3.1), it is clear that there must be a logarithmic divergence. In fact, we see below that the main cause of the divergence must be due to the shape of the Feynman propagator. This is related to the fact that the Feynman propagator does not satisfy the very important constraint which should arise from the photon equation of motion.

3.2 Propagator and Polarization Vector

Here we should briefly examine the Feynman propagator. This is related to the following quantity $\langle 0|T\{A^\mu(x_1)A^\nu(x_2)\}|0\rangle$ which can easily be calculated to be

$$\langle 0|T\{A^\mu(x_1)A^\nu(x_2)\}|0\rangle = -i\int \frac{d^4k}{(2\pi)^4}\frac{e^{ik(x_1-x_2)}}{k^2-i\varepsilon} \times \sum_{\lambda=1}^{2} \epsilon^\mu_{\boldsymbol{k},\lambda}\epsilon^\nu_{\boldsymbol{k},\lambda}. \tag{3.3}$$

where $\epsilon^\mu_{\boldsymbol{k},\lambda}$ denotes the polarization vector of photon. In this case, however, we should obtain some conditions on the polarization vector in advance which should come from the photon equation of motion. Therefore, we should solve the photon equation of motion in order to obtain some constraint on the polarization vector.

3.2.1 Photon Equation of Motion and Polarization Vector

The Lagrange equation for the free vector potential A^μ can be written as

$$\partial_\mu F^{\mu\nu} = 0, \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{3.4}$$

This is the most important equation of motion of photon which must be solved before the gauge fixing. This equation can be rewritten as

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = 0. \tag{3.5}$$

Now, we insert the following expression of the gauge field of A^μ

$$A^\mu(x) = \sum_{\boldsymbol{k}}\sum_{\lambda=1}^{2} \frac{1}{\sqrt{2V\omega_{\boldsymbol{k}}}}\epsilon^\mu(\boldsymbol{k},\lambda)\left[c_{\boldsymbol{k},\lambda}e^{-ikx} + c^\dagger_{\boldsymbol{k},\lambda}e^{ikx}\right] \tag{3.6}$$

into eq.(3.4), and we obtain the constraint equation for the polarization vector $\epsilon^\mu(\boldsymbol{k},\lambda)$ as

$$k^2\epsilon^\mu - (k_\nu\epsilon^\nu)k^\mu = 0. \tag{3.7}$$

Here, we should find the condition that there must be non-zero solution of the polarization vector $\epsilon^\mu(\boldsymbol{k}, \lambda)$ in eq.(3.7), and this necessary and sufficient condition is that the determinant of eq.(3.7) must vanish to zero. That is,

$$\det\{k^2 g^{\mu\nu} - k^\mu k^\nu\} = 0. \tag{3.8}$$

This equation can be immediately solved, and we find

$$k^2 = 0. \tag{3.9}$$

Therefore, we should put the solution of $k^2 = 0$ into eq.(3.7), and we find

$$k_\mu \epsilon^\mu = 0, \qquad \textbf{(Lorentz condition)} \tag{3.10}$$

which is the most important condition on the polarization vector ϵ^μ.

3.2.2 Feynman Propagator

In general, the propagator should be written as

$$D^{\mu\nu}(k) = A(k) \times \sum_{\lambda=1}^{2} \epsilon^\mu_{\boldsymbol{k},\lambda} \epsilon^\nu_{\boldsymbol{k},\lambda} \tag{3.11}$$

which can be understood from eq.(3.3). Therefore, the propagator $D^{\mu\nu}(k)$ should be proportional to the product of two polarization vectors. This means that the propagator should satisfy the following condition

$$k_\mu D^{\mu\nu}(k) = A(k) \times \sum_{\lambda=1}^{2} k_\mu \epsilon^\mu_{\boldsymbol{k},\lambda} \epsilon^\nu_{\boldsymbol{k},\lambda} = 0 \tag{3.12}$$

which is the constraint of the Lorentz condition. On the other hand, the Feynman propagator $D_F^{\mu\nu}(k)$ becomes

$$k_\mu D_F^{\mu\nu}(k) = -\frac{k^\nu}{k^2 - i\varepsilon} \neq 0 \tag{3.13}$$

and therefore, it cannot satisfy the most important condition which arises from the photon equation of motion. It is, by now, obvious that this should be a very important defect of the Feynman propagator.

3.3 Vertex Corrections of Weak Vector Bosons

In the case of lepton vertex corrections, the contribution from Z^0 boson should also be important. The calculation of vertex corrections can be carried out just in the same way as the photon case. The only difference between them should be the shape of the propagator. The propagator of Z^0 boson can be written as [3]

$$D^{\mu\nu}(k) = -\frac{g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}}{k^2 - M^2 - i\varepsilon}. \tag{3.14}$$

The important point is that this propagator can satisfy the Lorentz condition since

$$k_\mu D^{\mu\nu}(k) = -\frac{k^\nu - \frac{k^2 k^\nu}{k^2}}{k^2 - M^2 - i\varepsilon} = 0. \tag{3.15}$$

In this case, the vertex corrections $\Lambda^\rho(p,q)$ due to Z^0 boson becomes

$$\Lambda^\rho(p,q) = -ig_z^2 e \int \frac{d^4k}{(2\pi)^4} \left(\frac{g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}}{k^2 - M^2 - i\varepsilon} \right) \gamma_\mu \gamma^5 \frac{1}{\not{q} - \not{k} - m_\ell} \gamma^\rho \frac{1}{\not{p} - \not{k} - m_\ell} \gamma_\nu \gamma^5$$

where m_ℓ denotes the lepton mass. Now, one can convince oneself that this calculation should not give rise to any infinity since

$$\Lambda^\rho(p,p) = -ieg_z^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 2x dx \frac{\left(\gamma_\mu \not{k} \gamma^\rho \not{k} \gamma^\mu - \frac{\not{k}\not{k}\gamma^\rho \not{k}\not{k}}{k^2} \right)}{(k^2 - s - i\varepsilon)^3} = 0 \tag{3.16}$$

where $s = M^2(1-x) + m_e^2 x^2$ is defined.

3.4 Lepton $(g-2)$ from Weak Vector Boson

As the calculated result of the lepton vertex corrections, we first write the electron $(g-2)$ case. This is written as [3]

$$\left(\frac{g-2}{2} \right)_\mu \simeq \frac{7\alpha_z}{12\pi} \left(\frac{m_e}{M_z} \right)^2 \sim 10^{-13}$$

where M_z is the mass of Z^0 boson. α_z denotes the weak coupling constant of Z^0 boson with leptons. The correction for electron is very small indeed, and it is consistent with the experiment.

3.4.1 Muon $(g-2)$

On the other hand, the predicted value of vertex corrections of muon from Z^0 boson is given in [3]

$$\left(\frac{g-2}{2}\right)_\mu \simeq \frac{7\alpha_z}{12\pi}\left(\frac{m_\mu}{M_z}\right)^2 \simeq 8.6\times 10^{-10}.$$

This value of muon (g-2) can be compared to the recent observed value of Fermilab muon (g-2) experiment [9]. Surprisingly, this theoretical value turns out to be of the same order to the experimental number of muon $(g-2)$.

3.5 Electron-Electron Scattering

There must be a specific reason as to why people have been using the Feynman propagator even though some people knew that it is not a right one. In fact, there is a good reason why people have made use of the Feynman propagator. This is connected to the electron-electron scattering experiment which can be correctly described by the calculation with the Feynman propagator. As one can prove, the scattering T-matrix of electron-electron scattering using the Feynman propagator can reproduce the experimental cross section of electron-electron scattering.

Why is it that the two approaches should agree with each other ? There is a physical reason for that. In fact, we can prove that the scattering T-matrix with the Feynman propagator can be reduced to the one that is calculated from the correct propagator if we make use of the fact that the scattered electrons should be on the mass shell. Namely, these electrons satisfy the free Dirac equation, and this is the basic reason why the two scattering T-matrices agree with each other.

3.5.1 Scattering T-matrix

Here, we should write the scattering T-matrices which are calculated from both of the propagators.

• (a) Feynman Propagator

In this case, the scattering T-matrix becomes

$$T^{(F)} = -\frac{e^2}{q^2}\left[\bar{u}(p'_1)\gamma^0 u(p_1)\bar{u}(p'_2)\gamma^0 u(p_2) - \bar{u}(p'_1)\boldsymbol{\gamma} u(p_1)\cdot\bar{u}(p'_2)\boldsymbol{\gamma} u(p_2)\right]. \quad (3.17)$$

• (b) Correct Propagator

In this case, the T-matrix from the Coulomb part becomes

$$T^{(C)} = \frac{e^2}{\boldsymbol{q}^2}\bar{u}(p'_1)\gamma^0 u(p_1)\bar{u}(p'_2)\gamma^0 u(p_2). \quad (3.18)$$

On the other hand, the T-matrix from the vector potential can be written as

$$T^{(A)} = \frac{e^2}{q^2}\left[\bar{u}(p'_1)\boldsymbol{\gamma} u(p_1)\bar{u}(p'_2)\boldsymbol{\gamma} u(p_2) - \bar{u}(p'_1)\boldsymbol{\gamma}\cdot\boldsymbol{q}u(p_1)\frac{1}{\boldsymbol{q}^2}\bar{u}(p'_2)\boldsymbol{\gamma}\cdot\boldsymbol{q}u(p_2)\right].$$

Here, we should note that the following free Dirac equations

$$\begin{aligned}(\not{p}_1 - m_1)u(p_1) &= 0,\ \bar{u}(p'_1)(\not{p}'_1 - m_1) = 0,\\ (\not{p}_2 - m_2)u(p_2) &= 0,\ \bar{u}(p'_2)(\not{p}'_2 - m_2) = 0\end{aligned}$$

should hold. Therefore, the $T^{(A)}$ becomes

$$T^{(A)} = \frac{e^2}{q^2}\left[\bar{u}(p'_1)\boldsymbol{\gamma} u(p_1)\cdot\bar{u}(p'_2)\boldsymbol{\gamma} u(p_2) + \bar{u}(p'_1)\gamma^0 u(p_1)\frac{q_1^0 q_2^0}{\boldsymbol{q}^2}\bar{u}(p'_2)\gamma^0 u(p_2)\right].$$

Thus, the total T-matrix of $T^{(C)}$ and $T^{(A)}$ can be written as

$$T^{(C)} + T^{(A)} = -\frac{e^2}{q^2}\left[\bar{u}(p'_1)\gamma^0 u(p_1)\bar{u}(p'_2)\gamma^0 u(p_2) - \bar{u}(p'_1)\boldsymbol{\gamma} u(p_1)\cdot\bar{u}(p'_2)\boldsymbol{\gamma} u(p_2)\right] \quad (3.19)$$

which agrees with the one that is calculated from the Feynman propagator. The agreement is, of course, due to the fact that the scattered electrons are on the mass shell.

3.5.2 Loop Diagrams

Here, we write the calculations of diagrams with one loop, for reference.

(a) Feynman Propagator

The self-energy of fermion becomes

$$\Sigma^{(F)}(p) = -ie^2 \int \frac{d^4k}{(2\pi)^4} \gamma_\mu \frac{1}{\not{p} - \not{k} - m + i\varepsilon} \gamma^\mu \frac{1}{k^2 - i\varepsilon}. \tag{3.20}$$

(b) Correct Propagator

In this case, the fermion self-energy can be written as

$$\Sigma^{(A)}(p) = ie^2 \int \frac{d^4k}{(2\pi)^4} \gamma^a \frac{1}{\not{p} - \not{k} - m + i\varepsilon} \gamma^b \frac{\left(\delta^{ab} - \frac{k^a k^b}{\boldsymbol{k}^2}\right)}{k^2 - i\varepsilon} \tag{3.21}$$

which is completely different from eq.(3.20).

3.6 Negative Legacy

It may not be a very serious mistake that the Feynman propagator is applied to some physical processes with loops. However, this model calculation was taken to be a standard prescription to calculate the vertex corrections. In connection with gauge theory models, people claimed that only the gauge field theory must be a right theoretical scheme. This belief became accepted for most of theoretical physicists for a long time, and this must have prevented a healthy development of theoretical physics for quite some time. In this sense, the Feynman propagator left us a huge negative legacy for physics as a result, which is very unfortunate indeed.

However, if we take a proper propagator, quantum electrodynamics should be most reliable as a theoretical scheme. Therefore, it should be stressed that we should always consider solving the equation of motion for photon in order to obtain proper constraints on the polarization vector of photon.

Chapter 4

Dimensional Regularization of 't Hooft and Veltman

The self-energy of photon should have a quadratic divergence if it is calculated properly. But this is not a physical observables, and therefore, this quadratic divergence should have no physical effects at all. However, people thought that the renormalization scheme cannot work if the quadratic divergence exists in the self-energy of photon. Therefore, they invented some technique which can suppress the quadratic divergence term in some way or the other. In particular, 't Hooft and Veltman proposed a new regularization scheme in terms of the dimensional regularization. This method assumes that the four dimensional integral in momentum space should be made in a Euclid space, and then the dimension of momentum integral should be replaced by $4-\varepsilon$ instead of 4. In this case, they could obtain quite a strange result that the quadratic divergence seems to have disappeared. However, some careful examinations can prove that the disappearance of the quadratic divergence is simply because they employed a mathematical formula in a wrong way.

4.1 Vacuum Polarization

Here, we explain the vacuum polarization which is related to the self-energy of photon. By now, it has become clear that the physics of the vacuum polarization is misunderstood, that is, the vacuum polarization should have the quadratic divergence, but it is thrown away by hand. In this case, they assume so called "gauge condition" in order to throw away the quadratic divergence. However, we can easily prove that this gauge condition cannot hold at all since the calculation is carried out with a simple mistake in mathematics.

Here we explain briefly the physical meaning of gauge condition. The gauge condition is based on the assumption that the vacuum polarization tensor $\Pi^{\mu\nu}(k)$ must satisfy the following condition (gauge condition)

$$k_\mu \Pi^{\mu\nu}(k) = 0. \tag{4.1}$$

However, eq.(4.1) does not hold true, and below we will see it more in detail.

4.1.1 Gauge Condition and Disappearance of Infinity

Why is it that such a simple mistake could occur in physics? This is simply related to the careless variable change in the integral that involves some infinities. Here, it is shown how and why people claimed that they proved eq.(4.1). First, we rewrite $k_\mu \Pi^{\mu\nu}(k)$ as

$$k_\mu \Pi^{\mu\nu}(k) = ie^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\left(\frac{1}{\not{p} - \not{k} - m + i\varepsilon} - \frac{1}{\not{p} - m + i\varepsilon}\right)\gamma^\nu\right]. \tag{4.2}$$

At this point, people employ the following variable change in eq.(4.2)

$$q = p - k. \tag{4.3}$$

In this case, they thought they could obtain the following equation

$$k_\mu \Pi^{\mu\nu}(k) = ie^2 \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{\not{q} - m + i\varepsilon}\gamma^\nu\right] - ie^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{\not{p} - m + i\varepsilon}\gamma^\nu\right] = 0$$

and thus they claimed that the gauge condition of eq.(4.1) was proved. For a long time, most of the theoretical physicists seem to have believed this strange proof.

Below we discuss the above mistake by showing concrete examples as to why they made a mistake in the variable change in the integral that contains some infinities. This is simply connected to the fact that it looks as if the following equation for the infinity may hold

$$\infty + c = \infty. \tag{4.4}$$

But this is, of course, incorrect.

4.1.2 Variable Change in Infinite Integral

Here, we should consider the following integral Q

$$Q = \int_{-\infty}^{\infty} \left((x-a)^2 - x^2\right) dx. \tag{4.5}$$

In the above equation, if we replace the variable x by $x' = x - a$, then we obtain

$$Q = \int_{-\infty}^{\infty} \left(x'^2 dx' - x^2 dx\right) = 0. \tag{4.6}$$

Therefore, it looks the quantity Q is zero. However, if we calculate the integral properly, then we obtain

$$Q = \int_{-\infty}^{\infty} \left((x-a)^2 - x^2\right) dx = \int_{-\infty}^{\infty} \left(a^2 - 2ax\right) dx = 2a^2 \times \infty \tag{4.7}$$

and Q is infinite. It is clear why we made a mistake in the calculation of Q. When we change the variable from x to $x' = x - a$, we should modify the integral range accordingly. This can be seen as

$$Q = \lim_{\Lambda\to\infty} \int_{-\Lambda}^{\Lambda} \left((x-a)^2 - x^2\right) dx = \lim_{\Lambda\to\infty} \left[\int_{-\Lambda-a}^{\Lambda-a} x'^2 dx' - \int_{-\Lambda}^{\Lambda} x^2 dx\right] = \lim_{\Lambda\to\infty} 2a^2\Lambda$$

and thus the value of Q should be infinite. Therefore, we should be very careful for calculating the integral which becomes infinite.

4.2 Dimensional Regularization

't Hooft and Veltman proposed a new but strange method of calculating the momentum integral in four dimensions [11]. In the four momentum integral, they introduced the integral dimension of $4-\varepsilon$ instead of 4, where ε is infinitesimally small number. Using this new method, they claimed that the quadratic divergence in the vacuum polarization should disappear. However, the reason why the divergence vanished is, by now, clear and this is simply because they employed a wrong mathematical formula.

Indeed, they should recover the result of $\varepsilon = 0$ limit in the dimensional regularization technique, but they could not obtain the original result.

4.2.1 Mathematical Mistake in Dimensional Regularization

Here, we should examine the dimensional regularization method. As a simplified example, we consider the following integral

$$I_n = \int d^D p \frac{1}{(p^2+a^2)^n}, \qquad D = 4 - \varepsilon \tag{4.8}$$

where a is a cpnstant, and n is an integer. For the vacuum polarization, the corresponding n should be $n = 1$, and thus we consider the case of $n = 1$. Here, we define the evaluated result of the angle integration by Ω_D, and then the integral I_1 becomes

$$I_1 = \Omega_D \int_0^\infty dp \, \frac{p^{D-1}}{p^2+a^2}. \tag{4.9}$$

Obviously, this integral should give a quadratic divergence. In this case, why do people claim that the quadratic divergence may disappear?

4.3 Integral in Complex Plane

In eq.(4.9), the calculated result should correspond to the pole contribution in the complex plane integral of p. However, the integral in complex plane must be examined as to whether the integral should converge at the infinite circle of radius R in the complex p plane. On the other hand, the integral of eq.(4.9) does not converge, and instead, it gives the infinity which is quadratically diverging. Therefore, if we take only the pole contribution, then we obtain a wrong result for the integral of eq.(4.9).

4.3.1 $n = 2$ Case

Now we consider the $n = 2$ case in I_n. In this case, the integral becomes

$$I_2 = \Omega_D \int_0^\infty p^{D-1} dp \frac{1}{(p^2+a^2)^2}. \tag{4.10}$$

This time, the integral at the infinite circle of radius R in the complex p plane becomes

$$R^{-\varepsilon} \to 0 \quad \text{with} \quad R \to \infty \tag{4.11}$$

and therefore, the integral is converging to zero. Thus, we can obtain the right result for the integral of I with $n = 2$ case from the pole contribution.

4.4 Mathematics and Physics in Regularization

Here we should discuss the meaning of regularizations in mathematics. This is not related to the regularization of 't Hooft and Veltman, but we may clarify some physical meaning of the regularizations, if it exists.

4.4.1 Regularization of Summation

Now we consider the following summation N_0

$$N_0 = \sum_{n=0}^{\infty} (-)^n \tag{4.12}$$

which cannot give any definite number. Here, we may regularize the sum which is defined as

$$N_\lambda = \sum_{n=0}^{\infty} (-)^n e^{-n\lambda} \tag{4.13}$$

where λ **denotes an infinitesimally small positive number. This equation of (4.13) can be calculated to be**

$$N_\lambda = \lim_{\lambda \to 0} \frac{1}{1+e^{-\lambda}} = \frac{1}{2} \tag{4.14}$$

which is a finite and definite number.

4.4.2 Mathematical Meaning of Regularization

Mathematically, eq.(4.13) and eq.(4.12) should be completely different from each other. This is clear since the value of λ **is infinitesimally small, but it is not zero. In this respect, it should be rather dangerous to apply the regularization method to physics. However, we cannot make any constructive comments on this problem since it is not related to nature.**

4.5 Negative Legacy

In some period of late 20 century, most of the field theory textbooks made use of the dimensional regularizations in order to explain regularization methods in theoretical physics. This must be related to the physical prejudice that the renormalization scheme must be the fundamental basis of quantum field theory.

Therefore, the dimensional regularization may well have been taken as some physical magic that can control the infinity in a simple way. However, it should be clear that there should be no magic in physics at all.

The concept of regularization in physics may well have left some negative legacy. In reality, there should be no infinity in the physical observables when the theoretical scheme is sound.

Chapter 5

Weinberg-Salam Standard Model

Most of the experiments in weak interactions can be basically reproduced well by the theory of conserved vector currents (CVC) [12]. However, this model is based on the current-current interactions, and therefore, there is a serious theoretical defect in this model. This is connected to the fact that this CVC theory has a quadratic divergence in the second order perturbative calculation. In reality, the weak coupling constant is very small, and thus the first order perturbative calculation can describe most of the experimental results quite well. Nevertheless, the defect is serious in terms of theoretical scheme, and thus it should be modified in some way or the other.

In late 1960's, Weinberg and Salam proposed a weak interaction theory which is based on the non-abelian gauge field theory [13, 14]. However, the constituent particles in the non-abelian gauge field theory cannot be physical observables, and thus this should not be a right theory to describe weak interactions. In addition, they made use of the Higgs mechanism to make gauge fields massive, and therefore, this model becomes just meaningless. However, they adjust their model by hand so as to reproduce the CVC theory, and therefore, if the Higgs mechanism is taken away and some parameters should be chosen in a proper way, then this model can naturally reproduce the experiments of weak processes.

5.1 Non-abelian Gauge Theory

It is long believed that the renormalization scheme in QED should be the fundamental theory in quantum field theory, and people thought that this success of renormalization scheme must have been due to the fact that QED is a gauge field theory. Unfortunately, however, this is unfounded, but nevertheless the belief that the gauge field theory must be a fundamental scheme in theoretical physics became accepted by most of the physicists at that time. Therefore, it was quite natural that the weak interaction theory should be constructed by some gauge field theory models. In addition, the weak processes should be considered to be a SU(2) group, and thus people employed the non-abelian gauge field theory.

At that time, however, people may well have thought that the non-abelian gauge theory should be similar to the U(1) gauge theory, apart from its complexity. In reality, however, the color charge of non-abelian gauge theory models should be gauge dependent, and thus, they should not be any physical observables al all. In this sense, the non-abelian gauge field theory is completely different from the U(1) gauge theory in that the constituents of the non-abelian gauge theory should not have any free particle states.

5.1.1 Mass of Gauge Field

In addition, the gauge fields should be massless while the mass that is required from weak interaction experiments should be considered very large. In fact, the masses of weak vector bosons are discovered in 1980's, and they should be around 80 GeV/c^2 or more. In any case, the attempt to build weak interaction theory models with gauge fields must have been obviously reckless.

5.2 Higgs Mechanism

In the Weinberg-Salam model, they started from the gauge field theory, and therefore, they had to consider some magics which may give a finite mass to the gauge fields. The magic is the Higgs mechanism which is too primitive in physics, and therefore, it should not be worthwhile explaining in this short note. Nevertheless, we should make a brief review of the Higgs mechanism.

5.2.1 Higgs Potential

The Lagrangian density of Higgs mechanism can be written as [15]

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi)^\dagger(D^\mu\phi) - U(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{5.1}$$

where $U(\phi),\ D^\mu,\ F^{\mu\nu}$ are defined as

$$U(\phi) = -\frac{1}{4}u_0\left(|\phi|^2 - \lambda^2\right)^2 \tag{5.2}$$

$$D^\mu = \partial^\mu + igA^\mu \tag{5.3}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{5.4}$$

Here, $u_0,\ \lambda$ are constants. In this short note, we only consider U(1) gauge field, for simplicity. This Lagrangian density is invariant under the following gauge transformation

$$A^\mu \rightarrow A^\mu + \partial^\mu\chi \tag{5.5}$$

$$\phi \rightarrow e^{-ig\chi}\phi. \tag{5.6}$$

The field potential $U(\phi)$ is called Higgs potential whose origin is unknown. Clearly, this Higgs potential is not a basic physical quantity at all. Further, even though this model looks like a gauge field theory model, the current of the Higgs field ϕ is not a conserved quantity, and thus the model is not well defined theoretically. This point should be discussed later more in detail.

5.3 Conserved and Non-conserved Currents

As noted above, we only consider the U(1) case since it is sufficient for the present discussion. In this case, the field equation for the scalar field ϕ becomes

$$\partial_\mu(\partial^\mu + igA^\mu)\phi = -u_0\phi\left(|\phi|^2 - \lambda^2\right) - igA_\mu(\partial^\mu + igA^\mu)\phi. \tag{5.7}$$

On the other hand, the field equation for the gauge field A_μ can be written as

$$\partial_\mu F^{\mu\nu} = gJ^\nu. \tag{5.8}$$

5.3.1 Conserved Current

Here, we note that the current J^μ of eq.(5.8) is defined as

$$J^\mu = \frac{i}{2}\left\{\phi^\dagger(\partial^\mu + igA^\mu)\phi - \phi(\partial^\mu - igA^\mu)\phi^\dagger\right\}. \tag{5.9}$$

In this case, the following equation

$$\partial_\mu J^\mu = 0 \tag{5.10}$$

holds, and thus J^μ is a conserved current. However, it should be noted that this current contains the vector potential A^μ.

5.3.2 Current of Complex Scalar Boson

Now the current of the complex scalar field J^μ_{CSB} can be written as

$$J^\mu_{CSB} = \frac{i}{2}\left\{\phi^\dagger(\partial^\mu\phi) - \phi(\partial^\mu\phi^\dagger)\right\}. \tag{5.11}$$

Here we note that this current J^μ_{CSB} is not gauge invariant under the following gauge transformation

$$\phi \to e^{-ig\chi}\phi \tag{5.12}$$

and thus the charge of the complex scalar field cannot be any physical observables [16]. Further, this current is not conserved since

$$\partial_\mu J^\mu_{CSB} \neq 0 \tag{5.13}$$

and thus, we prove that the complex scalar fields are not physical observables, even though they may couple to the electromagnetic fields by the minimal principle. Therefore, we see that the complex scalar field should not be physical at all.

5.4 Unitary Gauge

In the Higgs mechanism, the unitary gauge fixing is used. In this case, they fixed the gauge at the Lagrangian density level

$$\phi = \phi^\dagger. \tag{5.14}$$

In this case, the final Lagrangian density can be written as

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{4}u_0\left(|\lambda + \eta(x)|^2 - \lambda^2\right)^2 + \frac{1}{2}g^2(\lambda+\eta(x))^2 A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where the Higgs field is assumed to be

$$\phi = \phi^\dagger = \lambda + \eta(x). \tag{5.15}$$

5.4.1 Quadratic Divergence Term

In the Lagrangian of Higgs model, we define the third term as

$$\mathcal{L}_I = \frac{1}{2}g^2(\lambda+\eta(x))^2 A_\mu A^\mu. \tag{5.16}$$

In this case, since this interaction should be made of four point vertex type, it should give rise to the quadratic divergence term when the vector field A_μ is quantized. This means that the existence of the interaction term $\mathcal{L}_I$ should have some intrinsic defects within this theoretical scheme.

5.5 Spontaneous Symmetry Breaking

The Higgs model is based on the spontaneous symmetry breaking picture, but this Nambu model calculation of the spontaneous symmetry breaking is totally incorrect. Therefore, it should not be worth discussing it. Nevertheless, we should briefly describe the model here [3, 17].

5.5.1 Model of Spontaneous Symmetry Breaking

In order to discuss the chiral symmetry and its breaking, Nambu et al. proposed the following Lagrangian density [18]

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi + \frac{1}{2}G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\right] \tag{5.17}$$

where the fermion mass is set to zero. Now, eq.(5.17) is invariant under the following chiral transformation

$$\psi' = e^{i\alpha\gamma_5}\psi \tag{5.18}$$

and therefore, the axial vector current is conserved. Here, it should be noted that any models with massless fermion should have no connection to nature since the massless fermion model has no scale to describe any physical observables. In this sense, this model is physically meaningless.

5.5.2 Spontaneous Chiral Symmetry Breaking?

Nambu et al. made use of Bogoliubov transformation, and rewrote the Hamiltonian density [19]. Therefore, they find the term which looks like mass terms. Thus, they thought that the system may have broken the chiral symmetry. However, it is well known that the Bogoliubov transformation should induce some terms which correspond to the mass term, but this has nothing to do with the symmetry breaking. It is simply because of the approximation they employed. The Bogoliubov transformation should generate higher order terms, and if they should have considered them, they would not have found any symmetry breaking. Indeed, it is very difficult to calculate the higher order terms in the Bogoliubov transformation, but they should have evaluated these effects in some way or the other.

In addition, they claimed that they also found a massless boson which should be associated with the symmetry breaking. However, their claim of a massless boson is based on the pole of the S-matrix, but it is well-known that there is no theoretical foundation for their claim. That is, the pole of S-matrix should not correspond to any bound states.

5.6 Exact Solution of Chiral Symmetry Model

The Lagrangian density of the massless Thirring model can be written as [20]

$$\mathcal{L} = i\bar{\psi}\gamma_\mu \partial^\mu \psi - \frac{1}{2}\, g j^\mu j_\mu \tag{5.19}$$

where j_μ denotes a fermion current. This Lagrangian density is invariant under the following chiral transformation

$$\psi' = e^{i\alpha\gamma_5}\psi. \tag{5.20}$$

It is interesting to note that this Hamiltonian

$$\hat{H} = \int dx \left\{ -i \left(\psi_a^\dagger \frac{\partial}{\partial x}\, \psi_a - \psi_b^\dagger \frac{\partial}{\partial x}\, \psi_b \right) + 2g\psi_a^\dagger \psi_b^\dagger \psi_b \psi_a \right\} \tag{5.21}$$

can be solved exactly by the Bethe anzatz technique [21].

5.6.1 Exact Solution of Thirring Model Vacuum

The vacuum energy which is constructed from the Bethe ansatz solution of the Thirring model is described in terms of analytic expression [22]. This is quite important, and this analytic expression should give a decisive proof that the chiral symmetry is not broken at all. The discussion in detail can be found in reference [17].

5.6.2 Property of Thirring Model Vacuum by Exact Solution

Here we should briefly describe some properties of the exact vacuum energy in the massless Thirring model [17].

• Vacuum Energy in Thirring Model

The vacuum state, for sure, preserves the chiral symmetry. It should be quite interesting to note that the energy of the exact vacuum state turns out to be lower than that of the free state. Thus, this vacuum state should be realized, even though the two dimensional field theory model has no connection to real nature.

• Eigenvalue of Chiral Charge in Vacuum State

There is a chiral charge of the vacuum state which is the eigenvalue of the chiral operator. The chiral charge operator is written as

$$Q_5 = \int j_5^0(x)\, d^3r, \qquad (j_5^\mu = \bar{\psi}\gamma_\mu\gamma_5\psi). \tag{5.22}$$

The vacuum state of free state should have a right-left symmetry, and therefore, the chiral charge is zero. On the other hand, the chiral charge of the true vacuum state has ± 1.

• Note

The chiral charge is conserved, and thus its eigenvalue can vary from zero to some finite integer value. This finite value of the chiral charge has nothing to do with symmetry breaking. In old days, there were some misunderstandings in this context.

5.7 Negative Legacy

The standard model of Weinberg and Salam has some fundamental defects in the model construction. However, it is made so as to reproduce the CVC theory, and therefore, the final version of the model can reproduce the weak interaction experiments. In this respect, this model may not have left a big negative legacy in physics. However, both Higgs mechanism and the model calculation of spontaneous symmetry breaking should have been carried out in the very low standard, and thus their negative legacy must be serious in theoretical physics community.

Even at the present day, CERN seems to continue the search experiment of Higgs particle, and this negative legacy to the experimental physics must be non-trivial. It may take a long time to cure this scar.

Chapter 6

Misunderstanding of Quark Confinement

Quantum chromodynamics is the non-abelian gauge field theory, and it cannot be solved in the perturbation theory since the free Lagrangian densities of quarks and gluons are not gauge invariant. In the perturbation theory, we describe all the physical observables in terms of the properties of quarks and gluons, and if they are not related to physical observables, then there is no point of employing the perturbation theory.

Even until recently, people believed that the confinement of quarks must be due to the linearly rising potential, which is a dynamical confinement. However, this picture is wrong, and the confinement of quarks should be due to the kinematical effects where their color charges are gauge dependent. Therefore, their confinement should be absolute, which is indeed the consequence of the non-abelian nature of QCD.

The quark model itself must be a right scheme to describe hadrons. This belief is based on the observation that quarks should have the electromagnetic charges, and the electromagnetic current of quarks should be conserved in hadron states. In fact, the magnetic moments of proton and neutron can be well described by the SU(6) quark model since the spin operator of quarks should not critically depend on the quark distribution inside baryon.

6.1 Quantum Chromodynamics (QCD)

In this section, we explain some fundamental properties of QCD which should be important to understand the reason as to why QCD is difficult to handle.

6.1.1 Lagrangian Density of QCD

The Lagrangian density of QCD for quark fields ψ with $SU(N_c)$ colors is described as [23]

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu - m_0)\psi - \frac{1}{2}\mathrm{Tr}\{G_{\mu\nu}G^{\mu\nu}\} \tag{6.1}$$

where $G_{\mu\nu}$ is written as

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]. \tag{6.2}$$

Here, the gluon field A_μ is given as

$$A_\mu = A_\mu^a T^a \equiv \sum_{a=1}^{N_c^2-1} A_\mu^a T^a \tag{6.3}$$

where T^a corresponds to the generator of $SU(N_c)$ group and satisfies the following commutation relations

$$[T^a, T^b] = iC^{abc}T^c. \tag{6.4}$$

C^{abc} denotes the structure constant of group generators. For $SU(2)$ case, the structure constant C^{abc} becomes just the anti-symmetric symbol ϵ_{abc}. In eq.(6.1), Tr { } means the trace of the group generators of $SU(N_c)$, and the generators T^a are normalized according to

$$\mathrm{Tr}\{T^a T^b\} = \frac{1}{2}\delta^{ab}. \tag{6.5}$$

This Lagrangian density is invariant under the following gauge transformation

$$\begin{aligned} \psi' &= (1 - ig\chi)\psi = (1 - igT^a\chi^a)\psi, \quad \text{with} \quad \chi = T^a\chi^a && (6.6) \\ A'^a_\mu &= A^a_\mu - gC^{abc}A^b_\mu\chi^c + \partial_\mu\chi^a && (6.7) \end{aligned}$$

where χ depends on space and time as $\chi = \chi(t, \boldsymbol{r})$ which is infinitesimally small.

6.2 Global Gauge Symmetry

The Lagrangian density of QCD does not have a global gauge invariance, in contrast to the QED or gravity theory [3]. Before going to the discussion of QCD, we briefly explain the global gauge transformation in QED, and this is written as

$$\psi' = e^{-i\alpha}\psi. \tag{6.8}$$

In QED, the Lagrangian density is invariant under the global gauge transformation of eq.(6.8). This invariance is very important since it leads to the current conservation of fermions.

On the other hand, the global gauge transformation of QCD can be written as

$$\psi' = e^{-i\alpha_b T_b}\psi. \tag{6.9}$$

In this case, the interaction Lagrangian density $\mathcal{L}_I$

$$\mathcal{L}_I = -g\bar{\psi}\gamma^\mu A_\mu^a T^a \psi \tag{6.10}$$

is not invariant under the gauge transformation of eq.(6.9) since the interaction Lagrangian density changes into

$$\mathcal{L}_I' = -g\bar{\psi}e^{i\alpha_b T_b}\gamma^\mu A_\mu^a T^a e^{-i\alpha_c T_c}\psi \neq \mathcal{L}_I. \tag{6.11}$$

Obviously, this is due to the non-abelian nature of SU(3) color.

6.3 Color Charge of Quarks

The Lagrangian density of QCD [eq.(6.1)] is invariant under the local gauge transformation. However, the color charges for quark state $[\psi]$ and gluon state $[A_\mu]$ are not gauge invariant. Here we should write the color current of quarks j_μ^b

$$j_\mu^b = \bar{\psi}\gamma^\mu T^b \psi. \tag{6.12}$$

Under the following gauge transformation

$$\psi' = (1 - igT^a\chi^a)\psi \tag{6.13}$$

the quark color current j_μ^b changes into

$$j'^b_\mu = \bar\psi'\gamma^\mu T^b\psi' = \bar\psi(1+igT^a\chi^a)\gamma^\mu T^b(1-igT^a\chi^a)\psi \quad (6.14)$$
$$\neq \bar\psi\gamma^\mu T^b\psi \quad (6.15)$$

and thus, the quark color current is not invariant under the local gauge transformation. This is quite important since the color charge of quarks should not be physical observables. In fact, the color current of quarks is not conserved.

6.3.1 Quark Confinement

The color charges of quarks should depend on time, and therefore, they are not physical observables. In fact, this is directly connected to the confinement of quarks. This means that quarks are confined kinematically, not dynamically, and therefore, the confinement of quarks must be absolute.

6.4 Gauge Dependence of Free Lagrangian Density

The proof that the free Lagrangian density of QCD should depend on the gauge is not very difficult. However, it is only recent that this point is realized and confirmed, and this is quite unfortunate indeed.

6.4.1 Perturbation Cannot Be Defined!

The fact that quarks and gluons should not have any free states must be very serious since this means that there is no way to evaluate the perturbation theory. The only way to obtain the energy eigenvalue of the system is to diagonalize the total Hamiltonian in some truncated space without any further approximations.

As in QED, the only basis to evaluate four dimensional field theory model is the perturbation theory. There is no other way to calculate the field theory model. In QED, we describe all the physical observables in terms of free electron and photon states, but in QCD, we do not know how we can calculate any physical observables at the present stage.

6.5 Negative Legacy

For a long time, people believed that quarks should be confined dynamically in terms of linearly rising potential, and this was a common sense in theoretical physics until recently. In reality, quarks should be confined kinematically due to the gauge dependence of their color charges, and thus, the confinement is absolute.

However, this wrong picture of quark confinement with the linearly rising potential may not have left so much serious negative legacy. In any case, there is no way to calculate any physical observables in QCD, and in this sense, it should be very difficult to expect any reasonable progress in theoretical treatment of QCD in future.

At this point, we should stress the importance of quark model. There must be sufficiently large evidences as to why people believe and accept the concept of quark model. The most important point is that quarks should have the electromagnetic charge, and thus the electromagnetic current of quarks should be conserved inside hadrons. For example, proton is composed of [u,u,d] quarks where this u-quark should have the charge of $\frac{2}{3}e$ while the charge of d-quark is $-\frac{1}{3}e$. In fact, if we calculate the magnetic moment ratio $[R = \frac{\mu_p}{\mu_n}]$ of proton and neutron, then we find an excellent agreement between theory $[R_{\text{theo}} = -1.5]$ and experiment $[R_{\text{exp}} = -1.46]$. Therefore, as long as we see the behavior of quarks inside baryons, then this picture of quarks should be quite successful indeed.

Appendix A

Wave Propagation in Medium and Vacuum

The classical wave such as sound can propagate through medium. However, it cannot propagate in vacuum as is well known. This is, of course, clear since the classical wave is the chain of the oscillations of the medium due to the pressure on the density.

On the other hand, quantum wave including photon can propagate in vacuum since it is a particle. Here, we clarify the difference in propagation between classical and quantum waves. The most important point is that the classical wave should be always written in terms of real functions while photon or quantum wave should be described by the complex wave function of the following form of $e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ since it should be an eigenstate of the momentum operator.

A.1 What is Wave ?

The sound can propagate through medium such as air or water. The wave can be described by the following differential equation in one dimension

$$\frac{\partial^2 \phi(x,t)}{\partial t^2} = v^2 \frac{\partial^2 \phi(x,t)}{\partial x^2}. \tag{A.1}$$

where v denotes the speed of wave. The solution of eq.(A.1) is written as

$$\phi(x,t) = A_0 \sin(\omega t - kx) \tag{A.2}$$

where ω and k denote the frequency and the wave number of the wave, respectively. The dispersion relation of this wave can be written as

$$\omega = vk. \tag{A.3}$$

Here, it is important to note that the amplitude is written as the real function, in contrast to the free wave function of electron in quantum mechanics. In fact, the free wave function of electron can be written in one dimension as

$$\psi(x,t) = \frac{1}{\sqrt{V}} e^{i(\omega t - kx)} \tag{A.4}$$

which is a complex function. The electron can propagate by itself and there is no medium necessary for the electron motion.

What is the difference between the real wave amplitude and the complex wave function? Here, we clarify this point in a simple way.

A.1.1 Real Wave Function: Classical Wave

If the amplitude is real such as eq.(A.2), then it can only propagate in medium. This can be clearly seen since the energy of the wave can be transported in terms of the density oscillation which is a real as the physical quantity. In addition, the amplitude becomes zero at some point, and this is only possible when it corresponds to the oscillation of the medium. This means that the wave function of eq.(A.2) has nothing to do with the probability of wave object. Instead, if it is the oscillation of the medium, then it is easy to understand why one finds the zero point of the amplitude. The real amplitude is called a classical wave since it is indeed seen in the world of the classical physics.

A.1.2 Complex Wave Function: Quantum Wave

On the other hand, the free wave function of electron is a complex function, and there is no point where it can vanish to zero. Since this is just the wave function of electron, its probability of finding the wave must be always a finite constant which is, in this case, $\frac{1}{V}$ at any space point of volume V.

A.2 Classical Wave

The sound propagates in the air, and its propagation should be transported in terms of density wave. The amplitude of this wave can be written in terms of the real function as given in eq.(A.2). This is quite reasonable since the density wave should be described by the real physical quantity. Instead, this requires the existence of the medium (air), and the wave can propagate as long as the air exists. Here, the basic wave equation in one dimension is given in eq.(A.1), and it is similar to the wave equation in quantum mechanics, though it is a real differential equation.

A.2.1 Classical Waves Carry Their Energy?

In this case, a question may arise as to what is a physical quantity which is carried by the classical wave like sound. It seems natural that the wave carries its energy (or wave length). In fact, the transportation of the energy should be carried out by the compression of the density and successive oscillations of the medium. Therefore, this wave of sound is called compression wave.

A.2.2 Longitudinal and Transverse Waves

Here, we discuss the terminology of the longitudinal and transverse waves, even though we should not stress its physics too much since there is no special physical meaning.

• Longitudinal wave : The sound propagates as the compressional wave, and the oscillations should be always in the direction of the wave motion. In this case, it is called longitudinal wave. This wave can be easily understood since one can make a picture of the density wave.

• Transverse wave : On the other hand, if the motion of the oscillations is in the perpendicular to the direction of the wave motion, then it is called transverse wave. The tidal wave may be the transverse wave, but its description may not be very simple since the density change may not directly be related to the wave itself.

A.3 Quantum Wave

Photon and quantum wave should be quite different from the classical wave, and the quantum wave is a particle motion itself. No medium oscillation is involved. For example, a free electron moves with the velocity v in vacuum, and this motion is also called "wave". The reason why we call it wave is simply due to the fact that the equation of motion that describes electrons looks similar to the classical wave equation of motion. Further, the solution of the wave equation can be described as e^{ikx}, and thus it is the same as the wave behavior in terms of mathematics. But the physical meaning of quantum wave is completely different from the classical wave, and the quantum wave is just the particle motion which behaves as the probabilistic motion.

A.3.1 Quantum Wave (Electron Motion)

The wave function of a free electron can be described as

$$\psi(x,t) = \frac{1}{\sqrt{V}} e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} \tag{A.5}$$

which is a solution of the Schrödinger equation of a free electron,

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2m}\boldsymbol{\nabla}^2\psi \tag{A.6}$$

where $k = \sqrt{2m\omega}$, and V denotes the corresponding volume which does not appear in any physical observables. Since the Schrödinger equation is quite similar to the wave equation in a classical sense, one calls the solution of the Schrödinger equation as a wave. However, the physics of quantum wave should be understood in terms of quantum mechanics, and the relation to classical wave should not be stressed too much. That is, the quantum wave is completely different from the classical wave such as sound wave, and one should treat the quantum wave as it is. In addition, the behavior and physics of the classical wave are very complicated, and we do not fully understand the behavior of the classical wave since it involves many body problems in physics.

A.3.2 Photon

The electromagnetic wave is called photon which behaves like a particle and also like a wave. This photon can propagate in vacuum and thus it should be considered to be a particle. Photon can be described by the vector potential $\boldsymbol{A}$.

• $\boldsymbol{A}$ is real ! : However, the vector potential $\boldsymbol{A}$ which should correspond to photon is obviously a real function, and therefore, it cannot propagate like a particle. This can be easily seen since the free Hamiltonian of photon commutes with the momentum operator $\hat{\boldsymbol{p}} = -i\boldsymbol{\nabla}$, and therefore it is a simultaneous eigenstate of the Hamiltonian. Thus, the $\boldsymbol{A}$ should be an eigenstate of the momentum operator since the free state must be an eigenstate of momentum. However, any real function cannot be an eigenstate of the momentum operator, and thus the vector field in its present form cannot describe the free particle state of photon.

• Free solution of vector field : What should we do ? The only way of solving this puzzle is to quantize a photon field. First, the solution of $\boldsymbol{A}$ can be written as

$$\boldsymbol{A}(x) = \sum_{\boldsymbol{k},\lambda} \frac{1}{\sqrt{2\omega_k V}} \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} \left(c^\dagger_{\boldsymbol{k},\lambda} e^{-ikx} + c_{\boldsymbol{k},\lambda} e^{ikx} \right) \tag{A.7}$$

with $kx \equiv \omega_k t - \boldsymbol{k} \cdot \boldsymbol{r}$. Here, $\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}$ denotes the polarization vector which will be discussed later more in detail. As one sees, the vector field is indeed a real function.

• Quantization of vector field : Now we impose the following quantization conditions on $c^\dagger_{\boldsymbol{k},\lambda}$ and $c_{\boldsymbol{k},\lambda}$

$$[c_{\boldsymbol{k},\lambda},\ c^\dagger_{\boldsymbol{k}',\lambda'}] = \delta_{\boldsymbol{k},\boldsymbol{k}'}\delta_{\lambda,\lambda'}, \tag{A.8}$$

$$[c_{\boldsymbol{k},\lambda},\ c_{\boldsymbol{k}',\lambda'}] = 0, \qquad [c^\dagger_{\boldsymbol{k},\lambda},\ c^\dagger_{\boldsymbol{k}',\lambda'}] = 0. \tag{A.9}$$

In this case, $c^\dagger_{\boldsymbol{k},\lambda}$ and $c_{\boldsymbol{k},\lambda}$ become operators. Therefore, we should now prepare the Fock space on which they can operate. This can be defined as

$$c_{\boldsymbol{k},\lambda}|0\rangle = 0 \tag{A.10}$$

$$c^\dagger_{\boldsymbol{k},\lambda}|0\rangle = |\boldsymbol{k},\lambda\rangle \tag{A.11}$$

where $|0\rangle$ denotes the vacuum state of photon field. Therefore, if one operates the vector field on the vacuum state, then one obtains

$$\langle \boldsymbol{k},\lambda|\boldsymbol{A}(x)|0\rangle = \frac{1}{\sqrt{2\omega_k V}} \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} e^{-ikx}. \tag{A.12}$$

As one sees, this new state is indeed the eigenstate of the momentum operator and should correspond to the observables. Therefore, photon can be described only after the vector field is quantized. Thus, photon is a particle whose dispersion relation becomes

$$\omega_{\boldsymbol{k}} = |\boldsymbol{k}|. \tag{A.13}$$

A.4 Polarization Vector of Photon

Until recently, there is a serious misunderstanding for the polarization vector $\epsilon^{\mu}_{\boldsymbol{k},\lambda}$. This is related to the fact that the equation of motion for the polarization vector is not solved, and thus there is one condition missing in the determination of the polarization vector.

A.4.1 Equation of Motion for Polarization Vector

Now the equation of motion for $A^\mu = (A^0, \boldsymbol{A})$ without any source terms can be written from the Lagrange equation as

$$\partial_\mu F^{\mu\nu} = 0 \tag{A.14}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. This can be rewritten as

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = 0. \tag{A.15}$$

Now, the shape of the solution of this equation can be given as

$$A^\mu(x) = \sum_{\boldsymbol{k}} \sum_{\lambda} \frac{1}{\sqrt{2V\omega_{\boldsymbol{k}}}} \epsilon^{\mu}_{\boldsymbol{k},\lambda} \left[c_{\boldsymbol{k},\lambda} e^{-ikx} + c^{\dagger}_{\boldsymbol{k},\lambda} e^{ikx} \right] \tag{A.16}$$

and thus we insert it into eq.(A.15) and obtain

$$k^2 \epsilon^\mu - (k_\nu \epsilon^\nu) k^\mu = 0. \tag{A.17}$$

Now the condition that there should exist non-zero solution of $\epsilon^{\mu}_{\boldsymbol{k},\lambda}$ is obviously that the determinant of the matrix in the above equation should vanish to zero, namely

$$\det\{k^2 g^{\mu\nu} - k^\mu k^\nu\} = 0. \tag{A.18}$$

This leads to $k^2 = 0$, which means $k_0 \equiv \omega_{\boldsymbol{k}} = |\boldsymbol{k}|$. This is indeed a proper dispersion relation for photon.

A.4.2 Condition from Equation of Motion

Now we insert the condition of $k^2 = 0$ into eq.(A.17), and obtain

$$k_\mu \epsilon^\mu = 0 \tag{A.19}$$

which is a new constraint equation obtained from the basic equation of motion. Therefore, this condition (we call it "Lorentz condition") is most fundamental. It should be noted that the Lorentz gauge fixing is just the same as eq.(A.19). This means that the Lorentz gauge fixing is improper and forbidden for the case of no source term. In this sense, the best gauge fixing should be the Coulomb gauge

$$\boldsymbol{k} \cdot \boldsymbol{\epsilon} = 0 \tag{A.20}$$

from which one finds $\epsilon_0 = 0$, and this is indeed consistent with experiment.

• Number of freedom of polarization vector : Now we can understand the number of degrees of freedom of the polarization vector. The Lorentz condition $k_\mu \epsilon^\mu = 0$ should give one constraint on the polarization vector, and the Coulomb gauge fixing $\boldsymbol{k} \cdot \boldsymbol{\epsilon} = 0$ gives another constraint. Therefore, the polarization vector has only two degrees of freedom, which is indeed an experimental fact.

• State vector of photon : The state vector of photon is already discussed. But here we should rewrite it again. This is written as

$$\langle \boldsymbol{k}, \lambda | \boldsymbol{A}(x) | 0 \rangle = \frac{\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}}{\sqrt{2\omega_k V}} e^{-ikx}. \tag{A.21}$$

In this case, the polarization vector $\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda}$ has two components, and satisfies the following conditions

$$\boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} \cdot \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda'} = \delta_{\lambda,\lambda'}, \qquad \boldsymbol{k} \cdot \boldsymbol{\epsilon}_{\boldsymbol{k},\lambda} = 0. \tag{A.22}$$

A.4.3 Photon Is Transverse Wave ?

People often use the terminology of transverse photon. Is it a correct expression? By now, one can understand that the quantum wave behaves as a particle motion, and thus it has nothing to do with the oscillation of the medium. Therefore, it is meaningless to claim that photon is a transverse wave. The reason of this terminology may well come from the polarization vector $\epsilon_{\boldsymbol{k},\lambda}$ which is orthogonal to the direction of photon momentum. However, as one can see, the polarization vector is an intrinsic property of photon, and it does not depend on space coordinates.

• No rest frame of photon ! : In addition, there is no rest frame of photon, and therefore, one cannot discuss its intrinsic property unless one fixes the frame. Even if one says that the polarization vector is orthogonal to the direction of the photon momentum, one has to be careful in which frame one discusses this property.

In this respect, it should be difficult to claim that photon behaves like a transverse wave. Therefore, one sees that photon should be described as a massless particle which has two degrees of freedom with the behavior of a boson. There is no correspondence between classical waves and photon, and even more, there is no necessity of making analogy of photon with the classical waves.

A.5 Poynting Vector and Radiation

Here, we discuss the Poynting vector how it appears in physics, and show that it cannot propagate in vacuum at all, and thus it has nothing to do with radiations. Also, we present a brief description of the basic radiation mechanism how photon can be emitted.

A.5.1 Field Energy and Radiation of Photon

Before discussing the propagation of Poynting vector, we should first discuss the mechanism of the radiation of photon in terms of classical electrodynamics. The interaction Hamiltonian can be written as

$$H_I = -\int \boldsymbol{j} \cdot \boldsymbol{A}\, d^3r \tag{A.23}$$

which should be a starting point of all the discussions. Now, we make a time derivative of the interaction Hamiltonian and obtain

$$W \equiv \frac{dH_I}{dt} = -\int \left[\frac{\partial \boldsymbol{j}}{\partial t} \cdot \boldsymbol{A} + \boldsymbol{j} \cdot \frac{\partial \boldsymbol{A}}{\partial t} \right] d^3r. \tag{A.24}$$

Since we can safely set $A^0 = 0$ in this treatment, we find

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t}. \tag{A.25}$$

Therefore, we can rewrite eq.(A.24) as

$$W = \int \boldsymbol{j} \cdot \boldsymbol{E}\, d^3r - \int \frac{\partial \boldsymbol{j}}{\partial t} \cdot \boldsymbol{A}\, d^3r. \tag{A.26}$$

Defining the first term of eq.(A.24) as W_E, we can rewrite W_E as

$$W_E \equiv \int \boldsymbol{j} \cdot \boldsymbol{E}\, d^3r = -\frac{d}{dt} \left[\int \left(\frac{1}{2\mu_0} |\boldsymbol{B}|^2 + \frac{\varepsilon_0}{2} |\boldsymbol{E}|^2 \right) d^3r \right] - \int \boldsymbol{\nabla} \cdot \boldsymbol{S}\, d^3r \tag{A.27}$$

which is just the energy flow of electromagnetic fields.

A.5.2 Poynting Vector

Here, the last term of eq.(A.27) is Poynting vector $\boldsymbol{S}$ as defined by

$$\boldsymbol{S} \equiv \boldsymbol{E} \times \boldsymbol{B} \tag{A.28}$$

which is connected to the energy flow of the electromagnetic field. This Poynting vector is a conserved quantity, and thus it has nothing to do with the electromagnetic wave. In addition, it is a real quantity, and thus there is no way that it can propagate in vacuum. In addition, the Poynting vector cannot be a target of the field quantization, and thus it always remains classical since it is written in terms of $\boldsymbol{E}$ and $\boldsymbol{B}$. However, there is still some misunderstanding in the textbooks on Electromagnetism, and thus, one should be careful for the treatment of the Poynting vector.

• Exercise problem: Here, we present a simple exercise problem of circuit with condenser with C (disk radius of a and distance of d) and resistance with R. The electric potential difference V is set on the circuit. In this case, the equation for the circuit can be written as

$$V = R\frac{dQ}{dt} + \frac{Q}{C}.$$

This can be easily solved with the initial condition of $Q = 0$ at $t = 0$, and the solution becomes

$$Q = CV\left(1 - e^{-\frac{t}{RC}}\right).$$

Therefore, the electric current J becomes

$$J = \frac{dQ}{dt} = \frac{V}{R}e^{-\frac{t}{RC}}.$$

In this case, we find the electric field $\boldsymbol{E}$ and the displacement current $\boldsymbol{j}_d$

$$\boldsymbol{E} = \frac{Q}{\pi a^2}\boldsymbol{e}_z = \frac{VC}{\varepsilon_0 \pi a^2}\left(1 - e^{-\frac{t}{RC}}\right)\boldsymbol{e}_z \tag{A.29}$$

$$\boldsymbol{j}_d = \frac{\partial \boldsymbol{E}}{\partial t} = \frac{V}{R\pi a^2}e^{-\frac{t}{RC}}\boldsymbol{e}_z. \tag{A.30}$$

Thus, the magnetic field $\boldsymbol{B}$ becomes

$$\boldsymbol{B} = \frac{i_d\, r}{2}\boldsymbol{e}_\theta = \frac{r}{2\pi a^2 R}\; e^{-\frac{t}{RC}}\;\boldsymbol{e}_\theta$$

where $\int_C \boldsymbol{B}\cdot d\boldsymbol{r} = \mu_0 i_d \pi r^2$ is used. Therefore, the Poynting vector at the surface (with $r = a$) of the cylindrical space of the disk condenser becomes

$$\boldsymbol{S} = \boldsymbol{E}\times\boldsymbol{B} = -\frac{V^2}{2\pi a R d}\; e^{-\frac{t}{RC}}\left(1-e^{-\frac{t}{RC}}\right)\boldsymbol{e}_r.$$

It should be noted that the energy in the Poynting vector is always flowing into the cylindrical space. Therefore, the electric field energy is now accumulated in the cylindrical space. There is, of course, no electromagnetic wave radiation, and in fact, the Poynting vector is the flow of field energy, and has nothing to do with the radiation of photon.

A.5.3 Emission of Photon

The emission of photon should come from the second term of eq.(A.26) which can be defined as W_R, and thus

$$W_R = -\int \frac{\partial \boldsymbol{j}}{\partial t}\cdot \boldsymbol{A}\, d^3r. \tag{A.31}$$

In this case, we can calculate the $\frac{\partial \boldsymbol{j}}{\partial t}$ term by employing the Zeeman effect Hamiltonian with a uniform magnetic field of $\boldsymbol{B}_0$

$$H_Z = -\frac{e}{2m_e}\boldsymbol{\sigma}\cdot\boldsymbol{B}_0. \tag{A.32}$$

The relevant Schrödinger equation for electron with its mass m_e becomes

$$i\frac{\partial\psi}{\partial t} = -\frac{e}{2m_e}\boldsymbol{\sigma}\cdot\boldsymbol{B}_0\,\psi. \tag{A.33}$$

Therefore, we find

$$\frac{\partial \boldsymbol{j}}{\partial t} = \frac{e}{m_e}\left[\frac{\partial\psi^\dagger}{\partial t}\hat{\boldsymbol{p}}\psi + \psi^\dagger\hat{\boldsymbol{p}}\frac{\partial\psi}{\partial t}\right] = -\frac{e^2}{2m_e^2}\boldsymbol{\nabla}B_0(\boldsymbol{r}). \tag{A.34}$$

In order to obtain the photon emission, one should quantize the field $\boldsymbol{A}$ in eq.(A.31).

• Field quantization : The field quantization in electromagnetic interactions can be done only for the vector potential $\boldsymbol{A}$. The electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ are classical quantities which are defined before the field quantization.

A.6 Gravitational Wave

People often discuss the gravitational wave which is supposed to come from the Einstein equation. In this case, one sees that the equation for the metric tensor is all real, and thus the solution of this equation must be also real. Therefore, the gravitational wave, if at all exists, is a real function, and thus, it cannot propagate in vacuum unless one believes the aether hypothesis.

• No quantization of gravity : In addition, there is no physical meaning to quantize the metric tensor, and therefore, there is no chance that the gravitational wave propagates in vacuum.

A.6.1 General Relativity

Since we treat the gravitational wave, we should make a brief comment on the general relativity. Einstein invented the Einstein equation which is the second order differential equation for the metric tensor $g^{\mu\nu}$. The Einstein equation is written as

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi G_0 T^{\mu\nu} \tag{A.35}$$

where $R^{\mu\nu}$ is called Ricci tensor and is written in terms of second order differential of $g^{\mu\nu}$. $T^{\mu\nu}$ denotes the energy-momentum tensor which can be expressed by the distribution function of stars. Note that the energy-momentum tensor can be only defined when the distribution of stars is introduced. Classical particles cannot make the energy-momentum tensor since it is normally defined for quantum fields.

A question may arise as to why the general relativity can be related to the gravitational theory. This reason is simply because Einstein claimed that the gravitational Poisson equation should be derived from the general relativity at the weak gravitational limit. However, in his proof, he assumed the following strange equation

$$g^{00} \simeq 1 + 2\phi \tag{A.36}$$

where ϕ denotes the gravitational field. Because of this equation (A.36), he could derive the gravitational Poisson equation

$$\boldsymbol{\nabla}^2\phi(\boldsymbol{r}) = 4\pi G\rho(\boldsymbol{r}) \tag{A.37}$$

where G and ρ denote the gravitational constant and the density, respectively.

• Eq.(A.36) is correct ? :
Here, we show that eq.(A.36) is not only strange but simply incorrect. In order to do so, we should examine the physical meaning of the equation $g^{00} \simeq 1 + 2\phi$. We should notice that 1 (unity) in the right hand side of eq.(A.36) is a simple number. This is clear since the metric tensor is just the coordinate system itself. However, the gravitational field ϕ is a dynamical variable, and therefore this summation of two different categories is simply meaningless.

• No connection between general relativity and gravity :
By now it should be clear that the general relativity has nothing to do with gravity. It is a theory for the coordinate system (metric tensor), but it is not a theory to describe nature.

A rigorous proof that the metric tensor of $g^{\mu\nu}$ has nothing to do with gravity can be made in the following way. If we look at the Einstein equation [eq.(A.35)], this is a differential equation to determine the metric tensor that appears in the left hand side of eq.(A.35). On the other hand, the right hand side of eq.(A.35) consists of the energy momentum tensor which should be made from star distribution functions. However, the star distribution can be determined only after the distribution of stars should be solved with gravitational potential. Therefore, before determining the metric tensor, we must assume the gravitational force in advance, and thus the metric tensor should never become a function of gravity. Therefore, eq.(A.36) is simply incorrect.

A.6.2 Gravitational Wave

Since the general relativity has nothing to do with gravity, there is no chance to connect the gravitational wave to the general relativity. Further, as we see later, the gravity is not quantized, and therefore, there is no concept of gravitational wave in physics at all.

Appendix B

New Gravity Model

Quantum field theory is based on the free Dirac fields and four fundamental interactions. These are electromagnetic, weak, strong and gravitational interactions. In terms of coupling constant, the electromagnetic interaction must be a standard, and the strength of the coupling constant which is dimensionless is found to be

$$\alpha = \frac{1}{137}. \tag{B.1}$$

On the other hand, the strong interaction should be stronger by two orders of magnitude than the electromagnetic interaction while weak interaction must be weaker by a few orders of magnitude than the electromagnetic interaction. In this respect, the gravity is, by far, the weakest force among the four interactions. In fact, the gravity is by the order of $\sim 10^{-30}$ smaller than the electromagnetic interaction.

B.1 Introduction

Nevertheless, the gravity is very important in the universe for the formation of stars and galaxies since the force has a very long range, and it is always attractive. In fact, apart from strong interactions that should responsible for nuclear fusion in stars, the basic ingredients of forming stars and galaxies in the universe should be the gravitational interaction.

B.1.1 Why Gravity Has Large Effects on Star Formation?

The gravity is crucially important for the formation of stars even though the interaction strength is quite weak. There are two important aspects in the gravity when the stars should be formed. The first point is connected to the interaction range which is very long since it has the shape of $1/r$. The other point is that the gravity is always attractive and the strength of the force should be proportional to the masses of interacting objects. Therefore, as long as the corresponding body is massive, there should exist the attractive interactions from all other massive objects even though they are far away from each other. Because of the attractive nature, there should be no shielding in contrast to the electromagnetic cases.

B.1.2 Dirac Equation with Gravitational Potential

When the energy of a particle becomes as high as its mass, then we have to consider the relativistic equation of motion under the gravitational potential. In this case, the Newton equation is not appropriate for describing a relativistic motion, and thus, we have to find a new equation of motion. Since we know that the classical mechanics is derived from the Schrödinger equation, we should start from the relativistic equation in quantum mechanics. This is the Dirac equation, and therefore, we have to consider the Dirac equation with the gravitational interaction.

However, the Dirac equation with the gravitational potential has not been determined properly for a long time. This problem is connected to the ambiguity as to whether the gravitational potential should be taken as the fourth component of the vector type interaction or the mass term of scalar type interaction. This problem was not settled until recently, and thus, we should consider the gravitational field theory in some way or other. As will be discussed later, the new gravity model is, indeed, constructed in terms of a massless scalar field theory. Therefore, the corresponding Dirac equation with the gravitational potential is well established by now [3, 17].

B.2 Dirac Equation and Gravity

The Newton equation works very well under the gravitational potential, and indeed, the Kepler problem is best understood by solving the Newton equation.

• Ehrenfest Theorem :
This Newton equation itself is obtained from the Schrödinger equation by making some approximation such as Ehrenfest theorem. In this case, the time development of the expectation values of $\boldsymbol{r}$ and $\boldsymbol{p}$ in quantum mechanics lead to the Newton equation.

• Foldy-Wouthuysen Transformation :
The Schrödinger equation can be derived from the Dirac equation by making the Foldy-Wouthuysen transformation which is a unitary transformation. Therefore, the Dirac equation must be the starting point from which the Newton equation can be derived.

B.2.1 Dirac Equation and Gravitational Potential

As can be seen from the present discussion, it should be crucially important to have the Dirac equation with the gravitational potential properly taken into account. Otherwise, we cannot obtain the Newton equation with the gravitational potential. In other words, we should not start from the Newton equation with the gravitational potential since it is obtained only after some series of approximations should be properly made for quantum mechanics.

• Dirac Equation with Coulomb Potential :
Before going to the discussion of the Dirac equation with the gravity, we should first discuss the Dirac equation with the Coulomb potential of $V_c(r) = -\frac{Ze^2}{r}$. This is well-known and can be written as

$$\left(-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha} + m\beta - \frac{Ze^2}{r}\right)\Psi = E\Psi. \tag{B.2}$$

On the other hand, we should be careful in which way we put the gravitational potential of $V(r) = -\frac{GmM}{r}$ into the Dirac equation since there are two different ways, either the same way as the Coulomb case or putting the gravitational potential into the mass term.

• Dirac Equation with Gravitational Potential :
In fact, the right Dirac equation with the gravitational potential of $V(r) = -\frac{GmM}{r}$ **can be written by putting it into the scalar term as**

$$\left[-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha} + \left(m - \frac{G_0 mM}{r}\right)\beta\right]\Psi = E\Psi. \tag{B.3}$$

This is obtained from the field theoretical construction of the gravity model. By now, we see that the scalar type potential of gravity must be the right gravitational potential, and we should discuss it more in detail below.

B.3 New Gravity Model

When we wish to construct the theory of gravity, the first thing we should work out should be to find the framework in which the gravitational potential can be properly taken into account in the Dirac equation. Without doing this procedure, there should be no way to consider the theory of gravity. In fact, the Dirac equation for a particle with its mass m **in the gravitational potential can be written as**

$$\left[-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha} + \left(m - \frac{GmM}{r}\right)\beta\right]\Psi = E\Psi \tag{B.4}$$

where M **denotes the mass of the gravity center. In addition, if we make the non-relativistic reduction using the Foldy-Wouthuysen transformation, then we find the gravitational potential in classical mechanics**

$$V(r) = -\frac{GmM}{r} + \frac{1}{2mc^2}\left(\frac{GmM}{r}\right)^2 \tag{B.5}$$

where the second term of the right hand side should be the additional potential which appears as the relativistic effect. This additional potential of gravity is a new gravitational potential, and this must be a new discovery ever since nineteenth century. It turns out that this new potential can explain the problem of leap second of the earth revolution period which will be discussed later.

• **Rough Estimation of Relativistic Effect :**
Historically, the first check of the relativistic effect was done by Michelson-Morley using the velocity of the earth revolution which should be the fastest object relevant to the observed speed on the earth. The result of Michelson-Morley experiment showed that the speed of light is not affected by the earth revolution, and this leads to the concept of the relativity principle. The relativistic effect in this case is

$$\left(\frac{v}{c}\right)^2 \sim 1.0 \times 10^{-8} \tag{B.6}$$

where c and v denote the velocities of light and the earth revolution, respectively. It should be interesting to note that the leap second of the earth revolution period is found to be ($\Delta T/T \sim 2 \times 10^{-8}$) which is just the same order of magnitude as the relativistic effect.

B.3.1 Lagrangian Density

When we consider the theory of gravity, we should start from the scalar field theory since it gives always attractive interactions.

• **Lagrangian Density of Gravity :**
Here, we should write the Lagrangian density of a fermion field ψ interacting with the electromagnetic field A_μ and the gravitational field $\mathcal{G}$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - e\bar{\psi}\gamma^\mu A_\mu\psi - m(1+g\mathcal{G})\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{v\mu\nu} + \frac{1}{2}\partial_\mu\mathcal{G}\ \partial^\mu\mathcal{G} \tag{B.7}$$

where m denotes the fermion mass. The gravitational field $\mathcal{G}$ is a massless scalar field. The reason why people did not consider the scalar field for the gravity should be mainly because the scalar field should not be renormalizable. However, there is no necessity of the field quantization of the gravitational field, and thus, there is no divergence at all.

• **Gravity Cannot Be Gauge Theory :**
For a long time, people believed that the basic field theory must be a gauge theory, even though there is no foundation for this belief. Indeed, the gauge theory has both attractive and repulsive interactions, and therefore, it is clear that this model of gauge field theory should not be suitable for the gravity.

By now, it is known that only the gauge theory of quantum electrodynamics using the Feynman propagator should give rise to some divergences in the calculation of physical observables such as vertex corrections. In fact, there is no divergence for the vertex corrections which are calculated from the massive vector field theory [3].

B.3.2 Equation for Gravitational Field

From the Lagrangian density, we can obtain the equation for the gravitational field from the Lagrange equation. Here, we can safely make the static approximation for the equation of motion, and obtain the equation for the gravitational field $\mathcal{G}_0$ as

$$\boldsymbol{\nabla}^2 \mathcal{G}_0 = m g \rho_g \tag{B.8}$$

where $m\rho_g$ corresponds to the matter density. The coupling constant g is related to the gravitational constant G as

$$G = \frac{g^2}{4\pi}.$$

This equation eq.(B.8) is indeed the Poisson equation for gravity.

B.3.3 Dirac Equation with Gravitational Potential

From the Lagrangian density with gravity and electromagnetic interactions, we can derive the Dirac equation

$$\left[-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha} + m\beta\left(1 + g\mathcal{G}\right) - \frac{Ze^2}{r} \right] \Psi = E\Psi. \tag{B.9}$$

Further, in case the gravitational force is produced by nucleus with its mass of M, the Dirac equation becomes

$$\left[-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha} + \left(m - \frac{GmM}{r} \right)\beta - \frac{Ze^2}{r} \right] \Psi = E\Psi \tag{B.10}$$

which is just the equation discussed in the previous section.

B.3.4 Foldy-Wouthuysen Transformation of Dirac Hamiltonian

The Dirac equation with the gravitational interaction

$$\left[-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha}+\left(m-\frac{GmM}{r}\right)\beta\right]\Psi=E\Psi \tag{B.11}$$

can be reduced to the non-relativistic equation in quantum mechanics. This can be done in terms of Foldy-Wouthuysen transformation which is a unitary transformation. Therefore, the transformation procedure is very reliable indeed.

• Foldy-Wouthuysen Transformation :
Here, we start from the Hamiltonian with the gravitational potential

$$H=-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha}+\left(m-\frac{GmM}{r}\right)\beta. \tag{B.12}$$

This Hamiltonian can be rewritten in terms of the Foldy-Wouthuysen transformation which is somewhat a complicated and tedious procedure involved, though it can be done in a straightforward way [26]. In this case, the non-relativistic Hamiltonian should be obtained as

$$H=m+\frac{\boldsymbol{p}^2}{2m}-\frac{GmM}{r}+\frac{1}{2m^2}\frac{GmM}{r}\boldsymbol{p}^2-\frac{1}{2m^2}\frac{GMm}{r^3}(\boldsymbol{s}\cdot\boldsymbol{L}) \tag{B.13}$$

which is kept only up to the order of $\left(\frac{\boldsymbol{p}}{m}\right)^2\frac{GM}{r}$**.**

B.3.5 Classical Limit of Hamiltonian with Gravity

Here, we should calculate the classical equation of motion from the non-relativistic Hamiltonian in quantum mechanics. In this case, the Hamiltonian which is only relevant to the present discussion can be written as

$$H = \frac{\boldsymbol{p}^2}{2m} - \frac{GmM}{r} + \frac{1}{2m^2}\frac{GmM}{r}\boldsymbol{p}^2. \tag{B.14}$$

This can be reduced to the Newton equation by making the expectation values of operators in quantum theory in terms of the Ehrenfest theorem. In this case, we approximate the products by the factorization in the following way

$$\left\langle \frac{1}{2m^2}\frac{GmM}{r}\boldsymbol{p}^2 \right\rangle = \left\langle \frac{1}{2m^2}\frac{GmM}{r} \right\rangle \left\langle \boldsymbol{p}^2 \right\rangle \tag{B.15}$$

which must be a good approximation in the classical mechanics application. In addition, we make use of the Virial theorem

$$\left\langle \frac{\boldsymbol{p}^2}{m} \right\rangle = -\left\langle V \right\rangle . \tag{B.16}$$

Therefore, we finally obtain the following additional potential

$$V(r) = -\frac{GmM}{r} + \frac{1}{2mc^2}\left(\frac{GmM}{r}\right)^2 \tag{B.17}$$

which is a new gravitational potential in classical mechanics. The derivation of the additional potential is similar to the Zeeman effects in that both interactions appear in the non-relativistic reduction as the higher order terms of coupling constant.

B.4 Predictions of New Gravity Model

By now, a new gravity model is constructed, and as a byproduct, there appears the additional gravitational potential. This is a very small term, but its effect can be measurable. Indeed, this is the relativistic effect which becomes

$$\left(\frac{v}{c}\right)^2 \sim 1.0 \times 10^{-8} \tag{B.18}$$

for the earth revolution around the sun. On the other hand, the leap second of the earth revolution is found to be

$$\left(\frac{\Delta T}{T}\right) \sim 2 \times 10^{-8} \tag{B.19}$$

which is just the same order of magnitude as the relativistic effect. Therefore, as we see later, it is natural that the leap second value can be understood by the additional potential of the new gravity model.

B.4.1 Period Shifts in Additional Potential

In the new gravity model, there appears the additional potential in addition to the normal gravitational potential. In the case of the earth revolution around the sun, this potential is written as

$$V(r) = -\frac{GmM}{r} + \frac{1}{2mc^2}\left(\frac{GmM}{r}\right)^2 \tag{B.20}$$

where the second term is the additional potential [**3**]. Here, G and c denote the gravitational constant and the velocity of light, respectively. m and M correspond to the masses of the earth and the sun, respectively.

• Non-integrable Potential :
It should be important to note that the additional potential should be a non-integrable, and therefore, the treatment should be done in terms of the perturbation theory. In this case, the Newton equation with the perturbative procedure of the additional potential can be solved, and the period T of the revolution is written as

$$\omega T \simeq 2\pi(1 + 2\eta) \tag{B.21}$$

where η is given as

$$\eta = \frac{G^2M^2}{c^2R^4\omega^2}. \tag{B.22}$$

Here, R is the average radius of the earth orbit. The angular velocity ω is related to the period T by

$$\omega = \frac{2\pi}{T}. \tag{B.23}$$

The period shift due to the additional potential becomes

$$\frac{\Delta T}{T} = 2\eta \tag{B.24}$$

which is the delay of the period of the revolution [3, 17].

B.4.2 Period Shifts of Earth Revolution (Leap Second)

In the earth revolution, the orbit radius, the mass of the sun and the angular velocity can be written as

$$R = 1.496 \times 10^{11} \text{ m}, \quad M = 1.989 \times 10^{30} \text{ kg}, \quad \omega = 1.991 \times 10^{-7}. \tag{B.25}$$

In this case, the period shift becomes

$$\frac{\Delta T}{T} = 2\eta \simeq 1.981 \times 10^{-8}. \tag{B.26}$$

Therefore, the period of the earth revolution per year amounts to

$$\Delta T_{N.G.} = 0.621 \quad [\text{s/year}] \tag{B.27}$$

which is a delay. This suggests that the corrections must be necessary in terms of the leap second.

• Leap Second :
In fact, the leap second corrections have been made for more than 40 years. The first leap second correction started from June 1972, and for 40 years, people made corrections of 25 second. Therefore, the average leap second per year becomes

$$\Delta T_{N.G.}^{Obs} \simeq 0.625 \pm 0.013 \quad [\text{s/year}] \tag{B.28}$$

which agrees perfectly with the prediction of eq.(B.27).

• Definition of Newcomb Time :
Newcomb defined the time series of second in terms of the earth revolution period. However, the recent measurement of time in terms of atomic clock turns out to deviate from the Newcomb time [24]. This deviation should be due to the relativistic effects, and indeed this deviation can be understood by the additional potential of gravity.

B.4.3 Mercury Perihelion Shifts

For a long time, people believed that the Mercury perihelion shifts can be understood by the higher order effects of general relativity. However, it is proved that there should be no perihelion shifts for one period of the earth revolution.

Instead, there should be the Mercury perihelion shifts which may arise from the effects of other planets such as Jupiter if we can measure the perihelion shifts for some long period of revolutions. Concerning the Mercury perihelion shifts, however, the measurements as well as the calculations of the effects from other planets should be carried out more carefully. After the calculation of Newcomb in the 19 century, no careful calculation on the perihelion shifts has been done until now.

B.4.4 Retreat of Moon

The moon is also affected by the additional potential of gravity from the earth. The shifts of the moon orbit can be expressed just in the same way as the earth revolution. In this case, η can be written as

$$\eta = \frac{G^2M^2}{c^2R^4\omega^2}. \tag{B.29}$$

Here, R is the radius of the moon orbit. M and ω denote the mass of the earth and the angular velocity, respectively. They are written as

$$R = 3.844 \times 10^8 \quad \text{m}, \quad M = 5.974 \times 10^{24} \quad \text{kg}, \quad \omega = 2.725 \times 10^{-6} \tag{B.30}$$

Therefore, the period shift becomes

$$\frac{\Delta T}{T} = 2.14 \times 10^{-11}. \tag{B.31}$$

Now, we should carry out the calculation as to how the orbit can be shifted, and the shift of the angle can be written as

$$\Delta\theta = 4\pi\eta. \tag{B.32}$$

Thus, the orbit shift $\Delta\ell_m$ can be written as

$$\Delta\ell_m = R\Delta\theta \simeq 0.052 \quad \text{m} \tag{B.33}$$

and therefore, the shift per year becomes

$$\Delta\ell_{m\ (one\ year)} = \Delta\ell_m \times \frac{3.156 \times 10^7}{2.36 \times 10^6} \simeq 69.5 \quad \text{cm}. \tag{B.34}$$

• Calculated Results of Retreat of Moon :
Since the orbit of the moon is ellipse, the orbit shift can be seen as if it were retreated [27]. The orbit is described by

$$r = \frac{R}{1 + \varepsilon \cos\theta}. \tag{B.35}$$

In addition, the eccentricity is quite small ($\varepsilon = 0.055$) **and therefore, we can rewrite the above equation as**

$$r \simeq R(1 - \varepsilon \cos\theta). \tag{B.36}$$

Thus, the orbit shift Δr **at** $\theta \simeq \frac{\pi}{2}$ **becomes per year**

$$\Delta r \simeq R\Delta\theta\ \varepsilon \simeq \Delta\ell_{m\ (one\ year)}\ \varepsilon\ \simeq\ 3.8\ \ \mathrm{cm} \tag{B.37}$$

On the other hand, the observed value of the retreat shift of the moon orbit is

$$\Delta r_m^{obs} \simeq 3.8\ \ \mathrm{cm} \tag{B.38}$$

which agrees very well with the prediction.

• Retreat Shift is not Real! :
It should be noted that this observation is only possible by making use of the Doppler shift measurement. This is not a direct measurement of the moon orbit distance which is not possible due to the uncertainty of the accuracy of light velocity

$$c = (2.99792458\ \ \pm 0.000000012) \times 10^8\ \ \mathrm{cm/s}. \tag{B.39}$$

The accuracy of the orbit shift $\Delta r_m^{obs} \simeq 3.8\ \ \mathrm{cm}$ **is at the order of** 10^{-10} **while the light velocity is measured only up to** 10^{-8} **accuracy. This means that the shift of the orbit radius is just the instantaneous and apparent effect.**

B.5 Summary

The new gravity theory of eq.(B.7)) can naturally lead to the Dirac equation of eq.(B.3). This is very important in modern physics since we have now the Dirac equation with the gravitational potential properly taken into account. This Dirac equation can be reduced to the non-relativistic Hamiltonian which then gives rise to the Newton equation with the gravitational potential, and this new equation should contain a new gravitational potential as the additional potential.

• Massless Scalar Field :
The fact that the gravity is described by the massless scalar field can give rise to some important effects on the non-relativistic reduction. This is in contrast to the Coulomb case, but rather similar to the non-relativistic reduction of the vector potential case. In the non-relativistic reduction of the vector potential term in the Hamiltonian, we find new terms such as Zeeman effects or spin-orbit interactions. In the same way, in the non-relativistic reduction of the scalar potential term in the Hamiltonian, we find the new additional potential. In fact, this new additional potential can reproduce the leap second of the earth revolution.

• Inertial Mass and Gravitational Mass :
From experiments, it is known that the inertial mass and gravitational mass are just the same. This equivalence of two masses is taken to be one of the grounds in constructing the general relativity. On the other hand, this equivalence is derived as a natural consequence in the new gravity model. This is one of the strong reasons why this new gravity model is a correct theory of gravity.

Appendix C

Planet Effects on Mercury Perihelion

In this Appendix, we discuss the Mercury perihelion shifts which should come from the gravitational interactions between Mercury and other planets such as Jupiter or Saturn. This calculation can be carried out in the perturbation theory of the Newton dynamics, which is rather new to the classical mechanics. Here, we should compare the numerical results with those calculated by Newcomb in 1898.

C.1 Planet Effects on Mercury Perihelion

The motion of the other planets should affect on the Mercury orbits. However, this is the three body problems, and thus it is not easy to solve the equation of motion in an exact fashion. Here, we develop the perturbative treatment of the other planet motions. Suppose Mercury and the planet (Jupiter) are orbiting around the sun, and in this case, the Lagrangian can be written as

$$L = \frac{1}{2}m\dot{\boldsymbol{r}}^2 + \frac{GmM}{r} + \frac{1}{2}m_w\dot{\boldsymbol{r}_w}^2 + \frac{Gm_wM}{r_w} + \frac{Gmm_w}{|\boldsymbol{r}-\boldsymbol{r}_w|} \tag{C.1}$$

where $(m,\ \boldsymbol{r})$ and $(m_w,\ \boldsymbol{r}_w)$ denote the mass and coordinate of Mercury and the planet, respectively. The last term in the right side of eq.(C.1) is the gravitational potential between Mercury and the planet, and therefore, it should be much smaller than the gravitational force from the sun.

C.1.1 The Same Plane of Planet Motions

Here, we assume that the motion of Mercury and the planet must be in the same plane, and therefore we rewrite the Lagrangian in terms of polar coordinates in two dimensions

$$\begin{aligned} L &= \frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2) + \frac{GmM}{r} + \frac{1}{2}m_w(\dot{r_w}^2 + r_w^2\dot{\varphi_w}^2) + \frac{Gm_wM}{r_w} \\ &+ \frac{Gmm_w}{\sqrt{r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w)}}. \end{aligned} \tag{C.2}$$

In this case, the Lagrange equation for Mercury can be written as

$$m\ddot{r} = mr\dot{\varphi}^2 - \frac{GmM}{r^2} - \frac{Gmm_w(r - r_w\cos(\varphi - \varphi_w))}{(r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w))^{\frac{3}{2}}} \tag{C.3}$$

$$\frac{d}{dt}(mr^2\dot{\varphi}) = -\frac{GmMrr_w\sin(\varphi - \varphi_w))}{(r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w))^{\frac{3}{2}}} \tag{C.4}$$

$$m_w\ddot{r_w} = m_wr_w\dot{\varphi}^2 - \frac{Gm_wM}{r_w^2} - \frac{Gmm_w(r_w - r\cos(\varphi - \varphi_w))}{(r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w))^{\frac{3}{2}}} \tag{C.5}$$

$$\frac{d}{dt}(m_wr_w^2\dot{\varphi}) = -\frac{Gm_wMrr_w\sin(\varphi_w - \varphi))}{(r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w))^{\frac{3}{2}}}. \tag{C.6}$$

C.1.2 Motion of Mercury

If we ignore the interaction between Mercury and the planet, then the Mercury orbit is just given as the Kepler problem, and the equations of motion become

$$m\ddot{r} = mr\dot{\varphi}^2 - \frac{GmM}{r^2} \tag{C.7}$$

$$\frac{d}{dt}(mr^2\dot{\varphi}) = 0. \tag{C.8}$$

Here, the solution of the orbit trajectory is given as

$$r = \frac{A}{1 + \varepsilon\cos\varphi} \tag{C.9}$$

where A **and** ε **are written as**

$$A = \frac{\ell^2}{m\alpha}, \qquad \varepsilon = \sqrt{1 + \frac{2E\ell^2}{m\alpha^2}} \quad \textbf{with} \quad \alpha = GMm \tag{C.10}$$

which should be taken as the unperturbed solution of the revolution orbit.

C.2 Approximate Estimation of Planet Effects

Now we should make a perturbative calculation of the many body Kepler problem by assuming that the interaction between Mercury and the planet is sufficiently small. In this case, we can estimate the effects of other planets on the Mercury orbit. Here we write again the equation of motion for Mercury including the gravity from the other planet

$$\ddot{r} = \frac{\ell^2}{m^2 r^3} - \frac{GM}{r^2} - \frac{Gm_w(r - r_w\cos(\varphi - \varphi_w))}{(r^2 + r_w^2 - 2rr_w\cos(\varphi - \varphi_w))^{\frac{3}{2}}}. \tag{C.11}$$

Now we replace $r,\ r_w$ by the average orbit radius $R,\ R_w$ in the last term of the right side, and thus, the equation becomes

$$\ddot{r} = \frac{\ell^2}{m^2 r^3} - \frac{GM}{r^2} - \frac{Gm_w(R - R_w\cos(\varphi - \varphi_w))}{(R^2 + R_w^2 - 2RR_w\cos(\varphi - \varphi_w))^{\frac{3}{2}}}. \tag{C.12}$$

Below we present some approximate solution of eq.(C.12).

C.2.1 Legendre Expansion

First we define the last term of eq.(C.12) by F as

$$F(x) \equiv -\frac{Gm_w(R - R_w x)}{(R^2 + R_w^2 - 2RR_w x))^{\frac{3}{2}}}, \qquad \text{with} \quad x = \cos(\varphi - \varphi_w) \tag{C.13}$$

and we make the Legendre expansion

$$F(x) = -\frac{Gm_w R}{(R^2 + R_w^2)^{\frac{3}{2}}} + \frac{Gm_w R_w(R_w^2 - 2R^2)}{(R^2 + R_w^2)^{\frac{5}{2}}}\, x + \cdots \ . \tag{C.14}$$

Therefore we obtain the equation of motion

$$\ddot{r} = \frac{\ell^2}{m^2 r^3} - \frac{GM}{r^2} + \frac{Gm_w R_w(R_w^2 - 2R^2)}{(R^2 + R_w^2)^{\frac{5}{2}}}\cos(\varphi - \varphi_w) \tag{C.15}$$

where the constant term is irrelevant and thus we do not write it above.

C.2.2 Iteration Method

Now we employ the iteration method in order to solve eq.(C.15). First we make use of the solution of the Kepler problem

$$\varphi = \varphi^{(0)} + \omega t \tag{C.16}$$

$$\varphi_w = \varphi_w^{(0)} + \omega_w t \tag{C.17}$$

and thus eq.(C.15) becomes

$$\ddot{r} = \frac{\ell^2}{m^2 r^3} - \frac{GM}{r^2} + \frac{G m_w R_w (R_w^2 - 2R^2)}{(R^2 + R_w^2)^{\frac{5}{2}}} \cos(b + \beta t) \tag{C.18}$$

where b and β should be given as

$$b = \varphi^{(0)} - \varphi_w^{(0)}, \quad \beta = \omega - \omega_w. \tag{C.19}$$

C.2.3 Particular Solution

In order to solve eq.(C.18), we assume that the last term is sufficiently small and therefore r may be written in the following shape as

$$r = r^{(0)} + K \frac{G m_w R_w (R_w^2 - 2R^2)}{(R^2 + R_w^2)^{\frac{5}{2}}} \cos(b + \beta t) \tag{C.20}$$

where $r^{(0)}$ denotes the Kepler solution of $r^{(0)} = \frac{A}{1+\varepsilon \cos\varphi}$. Now we insert the solution of eq.(C.20) into eq.(C.18), and we find the solution of K as

$$K = -\frac{1}{\beta^2}. \tag{C.21}$$

Therefore, we obtain the approximate solution as

$$r = r^{(0)} - \frac{G m_w R_w (R_w^2 - 2R^2)}{(R^2 + R_w^2)^{\frac{5}{2}} \beta^2} \cos(b + \beta t). \tag{C.22}$$

C.3 Effects of Other Planets on Mercury Perihelion

Therefore we should put the Kepler solution for $r^{(0)}$ and thus the Mercury orbit can be written as

$$\begin{aligned} r &= \frac{A}{1+\varepsilon\cos\varphi} - \frac{Gm_w R_w (R_w^2 - 2R^2)}{(R^2+R_w^2)^{\frac{5}{2}}\beta^2}\cos(b+\beta t) \\ &\simeq \frac{A}{1+\varepsilon\cos\varphi + \frac{Gm_w R_w (R_w^2-2R^2)}{R(R^2+R_w^2)^{\frac{5}{2}}(\omega-\omega_w)^2}\cos(b+\beta t)} \end{aligned} \tag{C.23}$$

where we take $A \simeq R$ and also $\beta = \omega - \omega_w$. Here as for ε_w, we take

$$\varepsilon_w \equiv \frac{Gm_w}{RR_w^2(\omega-\omega_w)^2}\frac{\left(1-\frac{2R^2}{R_w^2}\right)}{\left(1+\frac{R^2}{R_w^2}\right)^{\frac{5}{2}}} \tag{C.24}$$

and using $b + \beta t = \varphi - \varphi_w$, we obtain

$$r \simeq \frac{A}{1+\varepsilon\cos\varphi + \varepsilon_w\cos(\varphi-\varphi_w)}. \tag{C.25}$$

This equation suggests that the Mercury perihelion may well be affected by the planet motions.

C.3.1 Numerical Evaluations

Now we calculate the Mercury perihelion shifts due to the planet motions such as Jupiter or Venus. In order to do so, we first rewrite $\varepsilon\cos\varphi + \varepsilon_w\cos(\varphi-\varphi_w)$ terms as

$$\varepsilon\cos\varphi + \varepsilon_w\cos(\varphi-\varphi_w) = c_1\cos\varphi + c_2\sin\varphi = \sqrt{c_1^2+c_2^2}\cos(\varphi+\delta) \tag{C.26}$$

where c_1 and c_2 are defined as

$$c_1 = \varepsilon + \varepsilon_w\cos\varphi_w \tag{C.27}$$

$$c_2 = \varepsilon_w\sin\varphi_w. \tag{C.28}$$

Here $\cos\delta$ can be written as

$$\cos\delta = \frac{c_1}{\sqrt{c_1^2+c_2^2}}. \tag{C.29}$$

Further, ε_w is much smaller than ε and thus eq.(C.29) becomes

$$\cos\delta = \frac{\varepsilon + \varepsilon_w \cos\varphi_w}{\sqrt{(\varepsilon + \varepsilon_w \cos\varphi_w)^2 + (\varepsilon_w \sin\varphi_w)^2}} \simeq 1 - \frac{1}{2}\left(\frac{\varepsilon_w}{\varepsilon}\right)^2 \sin^2\varphi_w. \qquad \text{(C.30)}$$

C.3.2 Average over One Period of Planet Motion

Now we should make the average over one period of planet motion and therefore we find

$$\frac{1}{2\pi}\int_0^{2\pi} \sin^2\varphi_w \, d\varphi_w = \frac{1}{2}. \qquad \text{(C.31)}$$

Thus, δ becomes

$$\begin{aligned} \delta \;&\simeq\; \frac{\varepsilon_w}{\sqrt{2}\,\varepsilon} \simeq \frac{1}{\sqrt{2}\,\varepsilon}\frac{GM}{R_w^2}\,\frac{1}{R(\omega-\omega_w)^2}\left(\frac{m_w}{M}\right)\frac{\left(1-\frac{2R^2}{R_w^2}\right)}{\left(1+\frac{R^2}{R_w^2}\right)^{\frac{5}{2}}} \\ &\simeq\; \frac{R_w\,\omega_w^2}{\sqrt{2}\,\varepsilon R\,(\omega-\omega_w)^2}\left(\frac{m_w}{M}\right)\frac{\left(1-\frac{2R^2}{R_w^2}\right)}{\left(1+\frac{R^2}{R_w^2}\right)^{\frac{5}{2}}} \end{aligned} \qquad \text{(C.32)}$$

where the planet orbits are taken to be just the circle, for simplicity.

C.3.3 Numerical Results

In order to calculate the effects of the planet motions on the δ, we first write the properties of planets in Table 1. Here, numbers are shown in units of the earth.

Table 1

	Mercury	Venus	Mars	Jupiter	Saturn	Earth	Sun
Orbit Radius	0.387	0.723	1.524	5.203	9.55	1.0	
Mass	0.055	0.815	0.107	317.8	95.2	1.0	332946.0
Period	0.241	0.615	1.881	11.86	29.5	1.0	
ω	4.15	1.626	0.532	0.0843	0.0339	1.0	

In Table 2, we present the calculations of the values δ for one hundred years of averaging and the calculations are compared with the calculated results by Newcomb.

Table 2 The values of δ for one hundred years

Planets	Venus	Earth	Mars	Jupiter	Saturn	Sum of Planets
δ by eq.(C.32)	49.7	27.4	0.77	32.1	1.14	111.1
δ by Newcomb	56.8	18.8	0.51	31.7	1.5	109.3

As one sees, the agreement between the present calculation and Newcomb results is surprisingly good [24]. Here we do not verify the calculation of Newcomb for the other planet effects on the Mercury perihelion shifts, and instead we simply employ his calculated results.

C.3.4 Comparison with Experiments

The observed values of the Mercury perihelion shifts are often quoted in some of the old textbooks. However, it should be very difficult to find some reliable numbers of the Mercury perihelion shifts since these values are determined for 100 years of observation period in 19 century. In this respect, the comparison between the calculation and observation should be a homework problem for readers.

Appendix D

No Time Delay in Moving Frame

From the Lorentz transformation of eq.(2.1), it looks that time in the moving frame deviates from the rest frame. However, t and x are variables, and thus, they are not directly related to physical observables. Below we examine whether the time difference of Δt in the Gedanken experiment should be delayed or not.

D.1 Incorrect Gedanken Experiment

Here we first explain the time difference Δt in the Gedanken experiment which is often discussed in the science history, though it is incorrect. First, we consider a train (moving frame) which is driving in the straight line with a constant velocity v. We assume that there should be big mirror wall in parallel to the straight line with its distance of ℓ.

D.1.1 Time Difference of Moving Frame from Rest Frame

First, an observer in the train emits laser beams against mirror wall. In this case, the observer in the train should not notice that the train is moving. Now this observer should detect the reflected laser beam and should measure the time difference ($2\Delta\tau$). In this case, we see

$$\ell = c\Delta\tau. \tag{D.1}$$

On the other hand, an observer at the rest frame should detect the laser beam which reflects and travels through the triangle trajectory. In this case, the time difference ($2\Delta t$) should be

$$\sqrt{(c\Delta t)^2 - \ell^2} = v\Delta t. \tag{D.2}$$

Therefore, we find

$$\sqrt{c^2 - v^2}\ \Delta t = c\Delta\tau \tag{D.3}$$

which gives us the following relation between the time differences of $\Delta\tau$ and Δt as

$$\Delta\tau = \sqrt{1 - \frac{v^2}{c^2}}\ \Delta t. \tag{D.4}$$

This suggests that the time difference in the moving frame seems to be somewhat smaller than that of the rest frame.

D.1.2 Time Difference of Rest Frame from Moving Frame

Now we should carry out the same type of Gedanken experiment from the observer at the moving frame. In this case, the rest frame is moving with the velocity of $-v$ for the observer of the moving frame. This can be easily seen if we solve the Lorentz transformation the other way around

$$x' = \gamma(x - vt), \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right), \qquad y' = y, \qquad z' = z. \tag{D.5}$$

Here we see that the rest frame is moving with its velocity of $(-v)$. But otherwise, everything is just the same as in the previous case. In this case, the observer in the rest frame emits laser beams against mirror wall, and the observer in the train should detect the reflected laser beam and should measure the time difference $(2\Delta ct)$. Thus, we find

$$\Delta t = \sqrt{1 - \frac{v^2}{c^2}}\ \Delta\tau. \tag{D.6}$$

D.1.3 Inconsistency of Time Difference

What is going on? The results of eqs. (D.4) and (D.6) contradict with each other. Since Δt and $\Delta\tau$ should be observables in the Gedanken experiment, there must be something wrong there.

D.2 Where is Incorrect Process in Gedanken Experiment?

What should be incorrect inductions in the Gedanken experiment? This can be easily seen if we look into eq. (D.2). After Δt, we took the coordinate of the train as $\Delta x' = \Delta x + v\Delta t$, which is wrong. The correct coordinate after Δt should be given by the Lorentz transformation as

$$\Delta x' = \gamma v \Delta t. \tag{D.7}$$

Thus, we should replace in the following way

$$v\Delta t \Longrightarrow \gamma v \Delta t, \qquad c\Delta t \Longrightarrow \gamma c \Delta t. \tag{D.8}$$

Therefore, eq. (D.4) becomes

$$\begin{aligned} \Delta\tau &= \sqrt{1-\frac{v^2}{c^2}} \times \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\,\Delta t \\ &= \Delta t. \end{aligned}$$

This clearly shows that there is no time delay, and there is no inconsistency. This is just all what we see from the relativity.

D.2.1 No Time Delay in Moving Frame!

From the Gedanken experiment, we see that there is no time delay in the moving frame as compared to the rest frame. This is quite reasonable since the relativity only states that any inertial frames should produce the same results of all physical observables.

In fact, the time interval is defined from the earth period T around the sun. It is, of course, clear that, in any inertial system, the period T is the same. Therefore, there is no time delay in any inertial system even if it is moving very fast.

D.3 Examples of Relativity

Here we should discuss possible observables when two inertial frames are involved in physical processes. It should be noted that this consideration is only related to the kinematics, and therefore, we cannot learn anything about dynamics of physical processes.

D.3.1 Doppler Effect of Light

When a star is moving away from the earth, then lights emitted from this star should be affected by the Lorentz transformation, and this is known as the Doppler effect. Let consider that a star is going away with its velocity v. The momentum p of light emitted at the star should become p' on the earth, and this relation is given by the Lorentz transformation as

$$p' = \gamma \left(p - \frac{vE}{c^2} \right) = \gamma \left(p - \frac{vp}{c} \right) = \frac{p\left(1 - \frac{v}{c}\right)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = p\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}. \tag{D.9}$$

This shows that the momentum of light is decreased. If we express the above relation in terms of wave length, then we obtain

$$\lambda' = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}\ \lambda. \tag{D.10}$$

Since the wave length of the observed light becomes longer, we call it "red shift". It should be noted that this naming has no physical meaning. It simply says that red light has a longer wave length than that of blue light. The physical reason of the Doppler shift is simply because the energy and momentum make a four dimensional vector and therefore this is affected by the Lorentz transformation.

D.3.2 Life Time of Muon Produced in Atmosphere

High energy cosmic ray (protons) may collide with atmospheric N_2 or other molecule and may produce muons with the mass of $m_\mu = 105.6$ MeV/c^2. The life time τ_0 of this lepton is around $\tau_0 \simeq 2 \times 10^{-6}$ s. Therefore, muon is unstable. Now a question is as to whether the life time of muon may be affected by the Lorentz transformation or not. This problem is often discussed in science history, but here we should present a right description of muon as to how far it can travel in the air.

Now the life time τ_0 can be written in terms of decay width Γ as

$$\tau_0 = \frac{\hbar}{\Gamma}. \tag{D.11}$$

Here we note that Γ is a Lorentz invariant quantity. Therefore, the life time is also Lorentz invariant, and thus the life time of muon should be the same in any inertial frame.

D.3.3 Travel Distance L of Muon

Now we should calculate the travel distance L of muon after it is created from the collision of protons with atmosphere. This can be evaluated from the Lorentz transformation $x = \gamma(x' + vt')$ as

$$L = \gamma v \tau_0. \tag{D.12}$$

Here we take, as an example, muon with its energy of 1 GeV. In this case, the velocity of muon can be approximated by light velocity of c. The Lorentz factor γ should be $\gamma \simeq 10.6$. Therefore, the value of L becomes

$$L = \gamma v \tau_0 = 10.6 \times 3 \times 10^8 \times 2 \times 10^{-6} \simeq 6.3 \text{ km} \tag{D.13}$$

which is longer by γ than $v\tau_0$. This indicates that the muon produced in the atmosphere may well have some chance to be observed on the earth.

D.3.4 Accelerator Experiment

Unstable particles created by the large accelerator should travel the distance which is given by eq. (D.12). This is longer by a factor of γ than $v\tau_0$, but it has nothing to do with the delay of life time of unstable particles. It is simply due to the Lorentz transformation.

Appendix E

New Evaluation of Rayleigh Scattering

Here we describe the theory of Rayleigh scattering in terms of quantum mechanics terminology. First, we briefly review the cross section of Rayleigh scattering which is obtained by the classical electrodynamics. However, it is shown that the cross section commonly used until now is ten orders of magnitude smaller than the cross section which is calculated by Compton scattering evaluation. Therefore, there must be something wrong with the old fashioned Rayleigh scattering evaluation.

E.1 Interaction of Photon with Electron

Photon should interact only with electron, and thus photon should scatter with electrons in atoms. The interaction Hamiltonian can be written as

$$H' = -\frac{e}{m}\boldsymbol{p}\cdot\boldsymbol{A}(x). \tag{E.1}$$

This expression is non-relativistic, but it is just the same as the relativistic case, apart from the spin part which is not included here. In this sense, this interaction of eq.(E.1) must be sufficient as long as we treat the interaction of photon with atomic electrons. Here, m denotes the mass of electron, and $\boldsymbol{p}$ is a momentum operator of electron. Also, $\boldsymbol{A}(x)$ denotes the vector potential which is given as

$$\boldsymbol{A}(x) = \sum_{\boldsymbol{k},\lambda} \frac{\boldsymbol{\epsilon}_k^\lambda}{\sqrt{2\omega_k V}} \left(c^\dagger_{\boldsymbol{k},\lambda} e^{ikx} + c_{\boldsymbol{k},\lambda} e^{-ikx} \right) \tag{E.2}$$

where $kx \equiv \omega_k t - \boldsymbol{k}\cdot\boldsymbol{r}$. Also, $c^\dagger_{\boldsymbol{k},\lambda}$ ans $c_{\boldsymbol{k},\lambda}$ should be the creation and annihilation operators of photon.

E.1.1 Scattering T-Matrix in Second Order Perturbation

We consider the scattering of photon with electrons in atoms. In this case, the scattering can be described in terms of the second order perturbation theory. The interaction Hamiltonian is given in eq.(E.1). The elastic scattering T-matrix of photon with electron in atom can be written as

$$\begin{aligned} T &= \sum_n \langle\phi_0|H'|\phi_n\rangle\langle\phi_n|H'|\phi_0\rangle \left(\frac{1}{E_n - E_i - k + i\varepsilon} + \frac{1}{E_n - E_i + k + i\varepsilon}\right) \\ &\simeq \left(\frac{e}{m\sqrt{2Vk}}\right)^2 \sum_{n,\lambda} \frac{2\langle\phi_0|(i\boldsymbol{k}\cdot\boldsymbol{r})(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)|\phi_n\rangle\langle\phi_n|(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)(-i\boldsymbol{k}\cdot\boldsymbol{r})|\phi_0\rangle}{E_n - E_i} \end{aligned}$$

where the initial state of the atom can be written as $|i\rangle = |\phi_0(\boldsymbol{r})\rangle$. Here, $\phi_0(\boldsymbol{r})$ denotes the ground state of electron in the atom. Now, we assume that $E_n - E_i \gg k$ which should be well satisfied in this discussion. Also, $|\phi_n\rangle$ denotes the $n-$th excited state of the atom. E_i and E_n denote the eigenvalues of the ground state and the $n-$th excited state in the atom, respectively. Here the photon state is approximated as

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \simeq 1 + i\boldsymbol{k}\cdot\boldsymbol{r} + \cdots \tag{E.3}$$

which is the long wave length approximation. Note that the wave length of visible lights must be $k \simeq 1.2 \times 10^5$ cm while the radius of stom should be $r \leq 1.0 \times 10^{-7}\ \mathrm{cm}^{-1}$. Thus, we find $kr \simeq 10^{-2}$ which is sufficiently small for the expansion in eq.(E.3).

E.1.2 Evaluation of Scattering T-Matrix

Here, we make the closure approximation, and assume

$$\Delta E \equiv E_n - E_i$$

where the n dependence is neglected in ΔE. In this case, we find

$$T = \left(\frac{e}{m\sqrt{2Vk}}\right)^2 \sum_\lambda \frac{2\langle\phi_0|(\boldsymbol{k}\cdot\boldsymbol{r})(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)(\boldsymbol{k}\cdot\boldsymbol{r})|\phi_0\rangle}{\Delta E} \simeq \left(\frac{e^2}{Vkm^2}\right)\frac{k^2}{\Delta E}.$$

E.2 Cross Section of Rayleigh Scattering

We should evaluate the cross section of Rayleigh scattering and estimate numerically the order of magnitude of the cross section. The differential cross section can be written in terms of the T-matrix as

$$\frac{d\sigma}{d\Omega} = 2\pi|T|^2 \frac{V}{(2\pi)^3} k^2 \left(\frac{V}{c}\right) = \frac{4\alpha^2 k^4}{m^4(\Delta E)^2} = \frac{r_0^2}{2}\left(\frac{\lambda_0}{\lambda}\right)^4 \tag{E.4}$$

where we introduce

$$\lambda = \frac{2\pi}{k}, \qquad r_0 = \frac{\alpha}{m} = 2.82 \times 10^{-13} \quad \text{cm.} \tag{E.5}$$

Also, we define λ_0

$$\lambda_0^4 \equiv \frac{8(2\pi)^4}{m^2(\Delta E)^2}. \tag{E.6}$$

As one sees, eq.(E.4) is just the cross section of Rayleigh scattering.

E.2.1 Numerical Value of λ_0

Now we should make a rough estimation of λ_0 **value. Here, we take** $m = 0.51$ **MeV/c**^{2} **and** $\Delta E \simeq 7$ **eV. In this case, we find**

$$\lambda_0 \simeq 1.1 \times 10^{-7} \text{ cm.} \tag{E.7}$$

For visible lights, we see $\lambda \simeq 4.5 \times 10^{-5}$ cm **and thus**

$$\left(\frac{\lambda_0}{\lambda}\right)^4 \simeq 3.6 \times 10^{-11} \tag{E.8}$$

which is extremely small. Thus, the Rayleigh scattering cross section becomes

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ray} \simeq 3.6 \times 10^{-11} \times \left(\frac{r_0^2}{2}\right). \tag{E.9}$$

This indicates that the Rayleigh scattering cannot be applied to nature.

E.3 Atomic Compton Scattering

Here we should present the calculation of the differential cross section of atomic Compton scattering where photon scatters with atomic electrons. In this case, it should involve many body effects in the scattering process as a result.

E.3.1 Evaluation of Scattering T-Matrix

The scattering T-matrix between photon and electrons in atoms can be calculated from the second order perturbation theory as

$$\begin{aligned} T_{A-Comp} &= \sum_n \langle\phi_0|H'|\phi_n\rangle\langle\phi_n|H'|\phi_0\rangle\left(\frac{1}{E_n-E_i-k+i\varepsilon}+\frac{1}{E_n-E_i+k+i\varepsilon}\right) \\ &= \left(\frac{e}{m\sqrt{2Vk}}\right)^2 \sum_{n,\lambda}\langle\phi_0|(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)|\phi_n\rangle\langle\phi_n|(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)|\phi_0\rangle\frac{2(E_n-E_i)}{(E_n-E_i)^2-k^2} \end{aligned} \tag{E.10}$$

which should generate the biggest contribution to the cross section of atomic Compton scattering. Here, we ignore the pole contribution, and further we make the long wave length approximation. In this case, the wave function of photon can be expanded as

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = 1 + i\boldsymbol{k}\cdot\boldsymbol{r} + \cdots \tag{E.11}$$

where we take only the first term in eq.(E.11). This approximation can be justified since we consider the scattering of visible lights with electrons.

The similar calculation was carried out in the textbook of Sakurai [25]. However, the treatment in this textbook contains the effects from the $\boldsymbol{A}^2$ term which should not be included. Further, the approximations employed there are quite rough and therefore the result of the cross section cannot be reliable, even though the textbook claimed that the shape of Rayleigh scattering cross section could be reproduced.

E.3.2 Closure Approximation and Virial Theorem

Now we rewrite the T-matrix of eq.(E.10) by making use of closure approximation and obtain

$$\begin{aligned} T_{A-Comp} &= \left(\frac{e}{m\sqrt{2Vk}}\right)^2 \frac{2}{\Delta E}\sum_\lambda \langle\phi_0|(\boldsymbol{p}\cdot\boldsymbol{\epsilon}_\lambda)^2|\phi_0\rangle F_k \\ &= \left(\frac{e^2}{2Vkm}\right)\left(\frac{4}{3\Delta E}\right)\left\langle\phi_0\left|\frac{\boldsymbol{p}^2}{2m}\right|\phi_0\right\rangle F_k \end{aligned} \tag{E.12}$$

where F_k is defined as

$$F_k \equiv \frac{1}{1-\left(\frac{k}{\Delta E}\right)^2}. \tag{E.13}$$

Further, by making use of the Virial theorem for the Coulomb potential of $[V_c(r) = -\frac{Ze^2}{r}]$, we find

$$\langle \phi_0 \left| \frac{\boldsymbol{p}^2}{2m} \right| \phi_0 \rangle = -\frac{1}{2} \langle \phi_0 \left| V_c(r) \right| \phi_0 \rangle = |E_0|. \tag{E.14}$$

Here E_0 denotes the eigenvalue of the ground state in atom. As an average value of ΔE, we take

$$\Delta E \simeq \frac{4}{3} |E_0|. \tag{E.15}$$

This is an approximation, but it should be rather reliable for the estimation of atomic excitations. Thus, we can write T_{Comp}

$$T_{A-Comp} = \frac{e^2}{2Vkm} F_k.$$

E.3.3 Cross Section of Atomic Compton Scattering

Now, the differential cross section of atomic Compton scattering can be written as

$$\left(\frac{d\sigma}{d\Omega}\right)_{A-Comp} = 2\pi \left(\frac{e^2}{2Vkm}\right)^2 \frac{V^2 k^2}{(2\pi)^3} |F_k|^2 = r_0^2 |F_k|^2. \tag{E.16}$$

In the case of visible lights, we may take

$$|F_k|^2 \simeq 1. \tag{E.17}$$

Therefore, we see that the differential cross section of atomic Compton scattering should be almost the same as the normal Compton scattering cross section of photon with electrons. Therefore, it is clear that the atomic Compton scattering is much larger than the Rayleigh scattering for visible lights.

E.3.4 Comparison of Atomic Compton Scattering and Rayleigh Scattering

Here we should compare two cross sections between atomic Compton scattering and Rayleigh scattering. It is easy to see where the difference between two scattering processes emerges. In order to see it, we write again the expansion of the photon wave function

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \simeq 1 + i\boldsymbol{k}\cdot\boldsymbol{r} + \cdots . \tag{E.18}$$

If we take the first term in eq.(E.18), then this corresponds to the atomic Compton scattering process. On the other hand, the second term of eq.(E.18) corresponds to the Rayleigh scattering process. In fact, the magnitude of $(\boldsymbol{k}\cdot\boldsymbol{r})$ should be much smaller than 1 for visible lights.

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